Wave Effects in Light
Doppler Effect

True of waves, too

lower frequency  

higher frequency


Doppler Effect - Sound

lower frequency

higher frequency
Doppler Demonstrations
Doppler Effect

Same effect in wavelength

- Source moving toward us: waves bunch up – shorter wavelength = *blueshift*
- Source moving away from us: waves stretch out – longer wavelength = *redshift*
Doppler Effect

Same effect in wavelength
- Source moving toward us: waves bunch up
  – shorter wavelength = blueshift
- Source moving away from us: waves stretch out
  – longer wavelength = redshift
Doppler Effect

\[ v_r \equiv \frac{dr}{dt} \equiv \text{radial velocity} \]

> 0 moving apart
< 0 moving toward
Doppler Effect

$$\lambda_{obs} = \lambda_{em} + s = \lambda_{em} + v_r P_{em}$$

$$P_{em} = 1/\nu_{em} = 1/(c/\lambda_{em}) = \lambda_{em}/c$$

Recall $$\nu = c/\lambda$$

$$\lambda_{obs} \approx \lambda_{em}(1 + v_r/c)$$

$$\nu_{obs} \approx \nu_{em}/(1 + v_r/c) \approx \nu_{em}(1 - v_r/c)$$

approximate, assuming : $$\nu << c$$
Doppler Effect (Cont.)

Time Dilation

\[ \Delta t_{obs} = \Delta t_{em} \sqrt{1 - \frac{v^2}{c^2}} \]

for radial motion \( v = v_r \)

\# of waves emitted = \( v \Delta t \)

\[ \nu_{obs} \Delta t_{obs} = \nu_{em} \Delta t_{em} \]

\[ \nu_{obs} = \nu_{em} \sqrt{1 - \frac{v_r^2}{c^2}} / \left(1 + \frac{v_r}{c}\right) \]

\[ \nu_{obs} = \nu_{em} \sqrt{\frac{1 - \frac{v_r}{c}}{1 + \frac{v_r}{c}}} \]

\[ \lambda_{obs} = \lambda_{em} \sqrt{\frac{1 + \frac{v_r}{c}}{1 - \frac{v_r}{c}}} \]
Redshift

\[ z \equiv \frac{\lambda_{\text{obs}} - \lambda_{\text{em}}}{\lambda_{\text{em}}} \] redshift

\[ z \approx \frac{v_r}{c} \text{ for } v \ll c \]

> 0 moving apart (redshift)

< 0 moving toward (blueshift)
Doppler Effect - Summary

\[ v_r \equiv \frac{dr}{dt} \equiv \text{radial velocity} \]

for radial motion

\[ v_{\text{obs}} = v_{em} \sqrt{\frac{1 - v_r / c}{1 + v_r / c}} \]

\[ \lambda_{\text{obs}} = \lambda_{em} \sqrt{\frac{1 + v_r / c}{1 - v_r / c}} \]

for \( v \ll c \), only depends on \( v_r \)

\[ v_{\text{obs}} \approx v_{em} \left(1 - \frac{v_r}{c}\right) \]

\[ \lambda_{\text{obs}} = \lambda_{em} \left(1 + \frac{v_r}{c}\right) \]
Doppler Effect - Summary

Frequency and wavelength of light changes if source or observer move
Observed Properties of Stars
ASTR 2110
Sarazin
Extrinsic Properties

Location
Motion
“kinematics”
Extrinsic Properties

Location

Use spherical coordinate system centered on Solar System

Two angles \((\theta, \phi)\)
Right Ascension \((\alpha = RA)\)
Declination \((\delta = Dec)\)

easy to measure accurately
Extrinsic Properties

Location

Use spherical coordinate system \((r, \theta, \phi)\) centered on Solar System

Two angles \((\theta, \phi)\)
- Right Ascension \((\alpha = RA)\)
- Declination \((\delta = Dec)\)

Radius \(r = \text{distance } d\)

Very hard to measure
Distances
Key Measurement Problem

Celestial Sphere
Astronomical objects are so far away, we have no ability to judge depth
Stars which appear close on the sky can be very far apart
Earth travels around Sun $\Rightarrow$ we view stars from different angles
The stars will appear to shift back and forth every year
Parallax
Parallax

Earth travels around Sun \(\Rightarrow\) we view stars from different angles

The stars will appear to shift back and forth every year

Effect decreases with increasing distance
Parallax

\[
\sin \pi \text{ (rad)} = \frac{\text{AU}}{d}, \quad \pi \text{ (rad)} \ll 1
\]

\[
\pi \text{ (rad)} = \frac{\text{AU}}{d}
\]

1 rad = 206,265" (arcsec)

\[
\pi'' = 206,265 \frac{\text{AU}}{d}
\]

Define 1 pc ≡ 206,265 AU = 3.09 × 10^{18} \text{ cm} = \text{parsec}

\[
\pi'' = \frac{1}{d_{pc}}, \quad d_{pc} = \frac{1}{\pi''}
\]
Parsec

Basic unit of distance in astronomy

\[ \text{parsec} = 2.06 \times 10^5 \text{ AU} = 3.09 \times 10^{18} \text{ cm} \]

= 3.26 light years

\[ \text{AU} = 1.50 \times 10^{13} \text{ cm} \]

Memorize
Parallax

Example: Nearest star Proxima Centauri
\[ d = 1.3 \text{ pc} \Rightarrow \pi = 0.77 \text{ arcsec} \]
Spatial resolution of telescopes, “seeing”
\[ \approx 1 \text{ arcsec} \]
So, very hard to center images < 0.01”
Can only determine parallax, distance for relatively nearby stars, \( d << 100 \text{ pc} \)
Extrinsic Properties

Location

Motion

Separate into two components

\[ v_r = \text{radial velocity} \]
\[ v_t = \text{tangential velocity} \]

\[ v_r \] changes distance
\[ v_t \] changes angle
Radial Velocity, Doppler Shift

\( v_r > 0 \) \rightarrow \) distance increases

Measured by Doppler Shift

\[ z \equiv \frac{\lambda_{obs} - \lambda_{em}}{\lambda_{em}} \]

redshift or Doppler shift

\( z > 0 \) \rightarrow moving away

for \( v << c \), (true of all Milky Way stars)

\[ z \approx \frac{v_r}{c} \]

\[ v_r = zc \]
Tangential Velocity Proper Motion

\[ v_t \Rightarrow RA, \ Dec \ (\text{angles}) \ \text{change} \]

\[
\begin{align*}
\Delta \theta &= \frac{s}{d} = \frac{v_t \Delta t}{d} \\
\tan \Delta \theta &= \frac{s}{d} = \frac{v_t \Delta t}{d}
\end{align*}
\]
Tangential Velocity: Proper Motion

\[ \tan \Delta \theta = \frac{s}{d} = \frac{v_t \Delta t}{d} , \Delta \theta << 1 \text{ radian}, \tan \Delta \theta \approx \Delta \theta \]

\[ \Delta \theta = \frac{s}{d} = \frac{v_t \Delta t}{d} \]

Define "proper motion" \( \mu \equiv \frac{d\theta}{dt} \rightarrow \frac{\Delta \theta}{\Delta t} \)

\[ \mu = \frac{v_t}{d} \text{ measured in "}/\text{yr"} \]
**Parallax + Proper Motion**

$v_{\text{orb}} (\text{Earth}) = 30 \text{ km/s}$

$v (\text{nearby stars}) \sim 20 \text{ km/s}$

Proper motion (1 year) \sim \text{parallax}

Parallax, periodic 1 year, east-west

Proper motion continuous
Distance Dependence

Doppler shift: \( z = \frac{v_r}{c} \)

Independent of distance!! Can do across whole Universe

Proper motion, parallax

\[ \pi = \frac{1}{d_{\text{pc}}} , \mu = \frac{v_r}{d}, \text{both} \sim \frac{1}{d} \]

Can only detect for nearby stars, < 100 pc
Intrinsic Properties of Stars
Luminosity and Flux

L = luminosity = energy / time from star (erg/s)

“Brightness” = flux F = energy / area / time at Earth (erg/cm²/s)

\[ F = \frac{L}{4\pi d^2} \]

“inverse square law”

\[ L = 4\pi d^2 F \]
Magnitudes

Hipparchus

1) Classified stars by brightness, brighter = 1\textsuperscript{st} magnitude, \ldots 6\textsuperscript{th} magnitude

2) Used eyes, human senses logarithmic

Magnitudes ➔ go backwards, logarithmic
Magnitudes

Write as 1.3 or 1.3 mag
5 mag = factor of 100 fainter
2.5 mag = factor of 10 fainter (1 order of magnitude)

Two stars, fluxes $F_1, F_2$

$$\frac{F_1}{F_2} = 10^{-\frac{(m_1-m_2)}{2.5}} = 10^{-0.4(m_1-m_2)}$$

$$m_1 - m_2 = -2.5 \log(\frac{F_1}{F_2})$$

($\log \equiv \log_{10}$, $\ln \equiv \log_e$)
Examples - 1

Two stars, a is twice as bright as b

\[ m_b = 10 \text{ mag.} \quad \text{What is} \quad m_a? \]

\[ F_a / F_b = 2 \]

\[ m_a - m_b = -2.5 \log \left( \frac{F_a}{F_b} \right) = -2.5 \log(2) \]

\[ = -2.5 \cdot 0.3 = -0.75 \]

\[ m_a = m_b - 0.75 = 10 - 0.75 = 9.25 \text{ mag} \]
Sirius is -1.5 mag, Castor is 1.6 mag. Which is brighter, and by what factor?

Brighter $\rightarrow$ smaller mag $\rightarrow$ Sirius

$\Delta m = -3.1$

$F_S / F_C = 10^{ -0.4 \cdot \Delta m } = 10^{ -0.4 \cdot ( -3.1) } = 10^{1.24} = 17$
Magnitudes
Stellar Colors
Stellar Colors
Stellar Colors

Star vary in color:

- Betalgeuse red, Sun yellow, Vega blue-white

Use filters to get flux in one color, compare
Color Filters for Observing

Johnson-Cousins Filter Response

Wavelength (μm)

0.3 0.5 0.7 0.9

U B V R I
Stellar Colors

Star vary in color:
Use filters to get flux in one color,
compare
Fluxes: $F_U$, $F_B$, $F_V$, ...
Magnitudes: $m_U = U$, $m_B = B$, $m_V = V$, ...
Stellar Colors

Color index, or just “color”

\[ \text{CI} = B - V, \ldots \]

Note: Given \( B - V \rightarrow \text{fixed } F_B / F_V \)

Just measures \underline{shape} of spectrum, not total flux

Independent of distance

Just gives color
Color mainly determined by temperature of stellar surface.

Stellar spectra ~ black body:

\[ \lambda_{\text{max}} \approx \frac{0.3 \text{ cm}}{T} \] (Wiens Law)

Higher T \(\rightarrow\) shorter \(\lambda\) \(\rightarrow\) bluer light.

Hot stars blue, B – V negative.
Cool stars red, B – V positive.
Stellar Temperatures

Range from

\[ T \approx 3000 \text{ K} \text{ to } 50,000 \text{ K} \]

(brown dwarfs, planets cooler, some stellar corpses hotter)
Stellar Temperatures

Star Colors

<table>
<thead>
<tr>
<th>Temp (°F)</th>
<th>O</th>
<th>B</th>
<th>A</th>
</tr>
</thead>
<tbody>
<tr>
<td>50,000 - 100,000</td>
<td>17,500 - 50,000</td>
<td>13,000 - 17,500</td>
<td></td>
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</table>

<table>
<thead>
<tr>
<th>F</th>
<th>G</th>
<th>K</th>
<th>M</th>
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</thead>
<tbody>
<tr>
<td>10,500 - 13,000</td>
<td>8,500 - 10,500</td>
<td>6,000 - 8,500</td>
<td>3,000 - 6,000</td>
</tr>
</tbody>
</table>
Bolometric Magnitude

Hard to measure all light from star, but

**Bolometric magnitude** ➔ magnitude based on **total** flux

\[ m_{\text{bol}} \]
Luminosity & Absolute Magnitude

Absolute magnitude $M = \text{magnitude if star moved to } d = 10 \text{ pc}$
Luminosity & Absolute Magnitude

\[ F = \frac{L}{4\pi d^2} \quad \frac{F}{F_{10}} = \left( \frac{10}{d_{pc}} \right)^2 \]

\[ m - M = -2.5 \log \left( \left( \frac{10}{d_{pc}} \right)^2 \right) = -5 \log \left( \frac{10}{d_{pc}} \right) \]

\[ m - M = 5 \log d_{pc} - 5 \equiv \text{distance modulus} \]
Luminosity & Absolute Magnitude

\[ m - M = 5 \log d_{pc} - 5 \equiv \text{distance modulus} \]

\[ M = m - 5 \log d_{pc} + 5 \]

\[ = m + 5 \log \pi'' + 5 \]
From distance to Sun (AU) and flux

\[ M_{\text{bol}}(\odot) = +4.74 \]

\[ L_{\odot} = 3.845 \times 10^{33} \text{ erg/s} \]
\[ = 3.845 \times 10^{26} \text{ J/s} = \text{W} \]

\[ M_{\text{bol}} = 4.74 - 2.5 \log(L/L_{\odot}) \]

Memorize
Stellar Luminosities

Very wide range

$10^{-4} \ L_\odot \leq L \leq 10^6 \ L_\odot$
Basic Numbers of Astronomy

Distance Scales

Astronomical Unit AU = $1.50 \times 10^{13}$ cm

Parsec pc = $3.09 \times 10^{18}$ cm

Solar Units

Solar Mass $M_\odot = 1.99 \times 10^{33}$ gm

Solar Radius $R_\odot = 6.96 \times 10^{10}$ cm

Solar Luminosity $L_\odot = 3.845 \times 10^{33}$ ergs/sec
Stellar Radii

For BB,

$L = (\text{area}) \sigma T^4 = 4\pi R^2 \sigma T^4$

$\sigma = 5.67 \times 10^{-5} \text{ erg/cm}^2/\text{s/K}^4$

Stefan-Boltzmann constant

Define effective temperature $T_{\text{eff}}$

such that $L = 4\pi R^2 \sigma T_{\text{eff}}^4$

If BB, then $T = T_{\text{eff}}$
Stellar Radii

Measure flux $F$, distance $d \rightarrow L$

Measure color $\rightarrow T_{\text{eff}}$ (estimate)

Solve for radius $R$
Stellar Radii

Find mainly three sets of radii

**Normal Stars**: “main sequence”, “dwarfs”

\[ 0.1 \, R_{\odot} < R < 20 \, R_{\odot} \]

sequence: small, cool, faint \(\rightarrow\) big, hot, bright

**Giants**: \( R > 100 \, R_{\odot} \sim \text{AU} \)

cool, \( T \sim 3000 \, \text{K} \)

**White Dwarfs**: \( R \approx 0.01 \, R_{\odot} \sim R(\text{Earth}) \)