Thermodynamics
ASTR 2110
Sarazin

Ludwig Boltzmann
Test #1

Monday, October 6, 11 - 11:50 am
Chem 305 (classroom)
You may not consult the text, your notes, or any other materials or any person
Bring pencils, paper, calculator

~2/3 Quantitative Problems (like homework problems)

~1/3 Qualitative Questions
   Multiple Choice, Short Answer, Fill In the Blank questions
   No essay questions
Test #1 (Cont.)

Material:

Chapters: Preface, 1-3, 5-7, 13, 19.3
Introduction, Coordinates & Time, Motions of Planets, Early Astronomy (Greeks – Renaissance), Kepler’s Laws, Newton’s Laws, Gravity, Light, Telescopes, Doppler Effect, Basic Stellar Properties, Binary Stars, the Sun

Homeworks 1-4

Know pc, AU, M☉, L☉, R☉
Test #1 (Cont.)

No problem set week of September 29 – October 5 to allow study for test

Review Session:
Discussion session
Friday, October 3, 3-4 pm
Thermodynamic Equilibrium

Thermodynamic equilibrium:
Unique state characterized by one number

Temperature T

In TE, populations of states generally vary as
population $\alpha \exp(-E / kT)$, where E is energy of state
Thermodynamic Equilibrium

Bound states:

\[
\frac{n_2}{n_1} = e^{-(E_2 - E_1)/kT} = e^{-\Delta E/kT}
\]

- \( kT \ll \Delta E \) every thing in ground state
- \( kT \gg \Delta E \) many states populated

Bound-free (ionization):

Saha equation
- \( kT \ll \text{IP} \) lower ionization
- \( kT \gg \text{IP} \) higher ionization
Thermodynamic Equilibrium

Free Particles:
Maxwellian distribution
\[ <E> = <\frac{1}{2} mv^2> = \frac{3}{2} kT \]
Hotter = faster moving atoms
Radiation in Thermodynamic Equilibrium

Planck or Blackbody Spectrum:

\[ I_v = B_v \equiv \frac{2h\nu^3}{c^2} \frac{1}{e^{h\nu/kT} - 1} \]

Rayleigh-Jeans Limit:

\[ h\nu \ll kT \]
\[ I_v \approx \frac{2\nu^2}{c^2} kT \]
Radiation in Thermodynamic Equilibrium

Wien’s Law:

\[ \lambda_{\text{max}} = \frac{c}{\nu_{\text{max}}} = 0.3 \text{ cm} \]

Hotter = shorter wavelength = bluer

Stefan-Boltzmann Law:

\[ L = \text{(area)} \ \sigma T^4 \]
\[ \sigma = 5.67 \times 10^{-5} \ \text{erg/(cm}^2 \ \text{• sec • K}^4) \]
Pressure Forces

Newton’s 3$^{rd}$ Law $\implies$
most forces cancel in bulk
But, new force appears
Pressure
pressure = force / area
Pressure Forces

\[ p = \text{momentum} \]

\[
\text{Force} = \frac{dp}{dt} = \frac{\Delta p}{\text{atom}} \frac{\text{atoms}}{\text{time}} \\
\frac{\Delta p}{\text{atom}} = 2 p_x = 2mv_x \\
\frac{\text{atoms}}{\text{time}} = nv_x (\text{area}) \\
n = \frac{\text{atoms}}{\text{volume}} \\
P = 2np_x v_x = 2nm \langle v_x^2 \rangle \\
P = nkT = \frac{\rho kT}{\mu m_p} \text{ ideal gas law}
Hydrostatic Equilibrium

Planets & Stars
Gravity is balanced by Pressure:

**Hydrostatic Equilibrium**

Consider a spherically symmetric object

\[ \vec{F} = -\frac{GM(r)m}{r^2} \hat{e}_r \]

\[ M(r) \equiv \text{mass interior to } r \]
Hydrostatic Equilibrium

For mass m, take a small cylinder
Area A
Height \( \Delta r \)
Filled with fluid
with mass density \( \rho \)

Assume pressure \( P \) and density only depend on radius \([P(r), \rho(r)]\), and that pressure decreases with radius.
Hydrostatic Equilibrium

Pressure forces \( = P \cdot \text{Area} \)

\[
\vec{F}_{\text{pres}} = P A \hat{e}_r - (P + \Delta P) A \hat{e}_r
\]

\( = (P - P - \Delta P) A \hat{e}_r \)

\( = -\Delta P A \hat{e}_r \)

\[
\vec{F}_{\text{grav}} = -\frac{G M(r) m}{r^2} \hat{e}_r
\]

\( M(r) \equiv \text{mass interior to } r \)

\( m = \rho \cdot \text{volume} = \rho A \Delta r \)
Hydrostatic Equilibrium

\[
\vec{F}_{\text{grav}} = -\frac{GM(r) \rho A \Delta r}{r^2} \hat{e}_r
\]

\[
\vec{F}_{\text{tot}} = \vec{F}_{\text{grav}} + \vec{F}_{\text{pres}} = 0
\]

\[
-\vec{F}_{\text{pres}} = \vec{F}_{\text{grav}}
\]

\[
\Delta P A \hat{e}_r = -\frac{GM(r) \rho A \Delta r}{r^2} \hat{e}_r
\]

\[
\frac{\Delta P}{\Delta r} = -\frac{GM(r) \rho}{r^2} \Rightarrow
\]

\[
\frac{dP}{dr} = -\frac{GM(r) \rho}{r^2}
\]
Stellar Spectra
ASTR 2110
Sarazin

Solar Spectrum
Theory of Stellar Atmospheres

Divide stars into

Atmosphere
- Narrow outer layer, 1 mean free path, $\tau \approx 1$
- Makes light we see

Interior
- Not directly observable
Theory of Stellar Atmospheres

Assume

1. Very thin
   \[ \Delta r \ll R \]
   Treat as flat plane

2. No energy sources
   \[ L_{in} = L_{out} = L \]

3. Static
   Forces balance, hydrostatic equilibrium
Theory of Stellar Atmospheres

\[ \frac{dP}{dr} = -\frac{GM(r)\rho}{r^2} \quad \text{Hydrostatic equilibrium} \]

\[ M(r) \equiv \text{mass interior to } r = M_* \]

\[ \rho \equiv \text{mass density } = \text{mass}/\text{volume} \]

\[ r \approx R_* \quad \text{constant, thin atmosphere} \]

\[ g_* \equiv \frac{GM_*}{R_*^2} \quad \text{surface gravity, constant} \]

\[ \frac{dP}{dr} = -g_* \rho \quad \text{Hydrostatic equilibrium} \]

\[ P = \frac{\rho kT}{\mu m_p} \quad \text{Pressure, ideal gas law} \]
Theory of Stellar Atmospheres

$L = \text{constant, thin atmosphere, flux important}$

$L = 4\pi R_*^2 \sigma T_{\text{eff}}^4$, \( F = L / (4\pi R_*^2) = \sigma T_{\text{eff}}^4 \) constant

Equation of Radiative Transfer

\[
\frac{dI_\nu}{dx} = -\alpha_\nu I_\nu + \epsilon_\nu \quad \text{absorption reduces, emission increases}
\]

\( \epsilon_\nu, \alpha_\nu \equiv \text{emissivity, opacity, depend on T,} \rho, \text{composition} \)
Theory of Stellar Atmospheres

\[ \frac{dP}{dr} = -g_\ast \rho \quad \text{Hydrostatic equilibrium} \]

\[ P = \frac{\rho kT}{\mu m_p} \quad \text{Pressure, ideal gas law} \]

\[ F = \frac{L}{(4\pi R_\ast^2)} = \sigma T_{\text{eff}}^4 \quad \text{constant} \]

\[ \frac{dI_\nu}{dx} = -\alpha_\nu I_\nu + \varepsilon_\nu \quad \text{Eqn. Radiative Transfer} \]
Theory of Stellar Atmospheres

\[
\frac{dP}{dr} = - g* \rho \quad \text{Hydrostatic equilibrium}
\]

\[
P = \frac{\rho kT}{\mu m_p} \quad \text{Pressure, ideal gas law}
\]

\[
F = \frac{L}{(4\pi R_*^2)} = \sigma T_{\text{eff}}^4 \quad \text{constant}
\]

\[
\frac{dI_\nu}{dx} = -\alpha_\nu I_\nu + \varepsilon_\nu \quad \text{Eqn. Radiative Transfer}
\]

Four unknowns: P, \(\rho\), T, \(I_\nu\) vs. \(r\)

Four equations \(\Rightarrow\) solvable
Inputs:

\[ F = \frac{L}{4\pi R_*^2} = \sigma T_{\text{eff}}^4 = \text{constant} \]

\[ g_* \]

\( \alpha_\nu, \varepsilon_\nu, \mu \) depend on \( \rho, T, \) \underline{composition} \n
\( R_* \) but only scales the fluxes
Stellar Spectra and Stellar Atmospheres

Determined by (in decreasing order of importance)

1. $T_{\text{eff}}$
2. $g^*$
3. Composition
Stellar Spectra and Stellar Atmospheres

Determined by (in decreasing order of importance)

1. $T_{\text{eff}}$
2. $g$
3. Composition
Solar Spectrum
Stellar Spectra

General result

Stellar spectra =

continuum emission + absorption lines

~blackbody
hotter → brighter, bluer

hotter →
molecular lines →
atomic lines →
ions (more and more ionized)
Stellar Spectra

hotter
cooler
Stellar Spectra

hotter

cooler
Stellar Spectra

ions
hotter

molecules
cooler
Stellar Spectra

The diagram illustrates the relative strengths of lines in stellar spectra at different temperatures. As the temperature increases from cooler to hotter, the strengths of specific elements, such as ionized helium, neutral helium, hydrogen, ionized metals, and neutral metals, change. The x-axis represents temperature, while the y-axis shows the relative strengths of these lines in the spectrum.
Spectral Classification

~1900, done by Annie Jump Cannon, assistant to Prof. E. C. Pickering at Harvard
Annie Jump Cannon
Annie Jump Cannon
Annie Jump Cannon
Annie Jump Cannon
Spectral Classification

~1900, before atomic theory, spectral lines hard to understand

Use Balmer lines of hydrogen
Stellar Spectra

Hβ hotter Hα

cooler
Spectral Classification

~1900, before atomic theory, spectral lines hard to understand
Use Balmer lines of hydrogen
From excited states

\[ n = 1 \]
\[ n = 2 \]
\[ n = 3 \]
\[ n = 4 \]
Stellar Spectra

- Hotter temperatures show stronger lines for ionized metals.
- Cooler temperatures show stronger lines for neutral metals.
- Temperature values range from 50,000 to 3,000 degrees Kelvin.

Key elements:
- Hydrogen: Strongest line in all temperatures.
- Helium: Strong in hotter temperatures.
- Metals: Strong in cooler temperatures.
Stellar Spectra

![Graph showing the relationship between temperature and Hα strength.](image)

- **Temperature** range: 3000 to 50000
- **Hα strength** increases with temperature up to a peak, then decreases.

Note: The graph illustrates the peak in Hα strength occurring around a temperature of 10000 K.
Stellar Spectra

**Graph:**
- X-axis: Temperature (3000 to 50000)
- Y-axis: $H\alpha$ strength

**Equation:**
$$\frac{n_2}{n_1} = \exp\left(-\frac{E}{kT}\right)$$

**Diagram:**
- Excitation and ionization of hydrogen
- Energy levels:
  - $H\beta$
  - $H\alpha$
  - $n = 4$
  - $n = 3$
  - $n = 2$
  - $n = 1$
Spectral Classification

~1900, before atomic theory, spectral lines hard to understand

Use Balmer lines of hydrogen

Alphabetical system (A - P) based mainly on strength of hydrogen Balmer lines

A = strongest
Stellar Spectra

![Graph showing Stellar Spectra with temperature on the x-axis and Hα strength on the y-axis. The graph has points labeled M, K, G, F, A, B, C, D, and E.](image)
Stellar Spectral Classes

\( T_{\text{eff}} \)

50,000 K \hspace{1cm} 2,000 K

O \hspace{0.5cm} B \hspace{0.5cm} A \hspace{0.5cm} F \hspace{0.5cm} G \hspace{0.5cm} K \hspace{0.5cm} M \hspace{0.5cm} L \hspace{0.5cm} T

Oh, be a fine \{ guy \}

\{ girl \} kiss me

Memorize

L, T = brown dwarfs
Stellar Spectra

- Hotter ions
- Cooler molecules
- Spectral Class

The graph shows the relative strengths of lines for different elements and their presence across various spectral classes.
### Spectral Types

<table>
<thead>
<tr>
<th>Spectral Type</th>
<th>Principal Characteristics</th>
<th>Spectral Criteria</th>
</tr>
</thead>
<tbody>
<tr>
<td>O</td>
<td>Hottest bluish-white stars; relatively few lines; He II dominates</td>
<td>Strong He II lines in absorption, sometimes emission; He I lines weak but increasing in strength from O5 to O9; hydrogen Balmer lines prominent but weak relative to later types; lines of Si IV, O III, N III, and C III</td>
</tr>
<tr>
<td>B</td>
<td>Hot bluish-white stars; more lines; He I dominates</td>
<td>He I lines dominate, with maximum strength at B2; He II lines virtually absent; hydrogen lines strengthening from B0 to B9; Mg II and Si II lines</td>
</tr>
<tr>
<td>A</td>
<td>White stars; ionized metal lines; hydrogen Balmer lines dominate</td>
<td>Hydrogen lines reach maximum strength at A0; lines of ionized metals (Fe II, Si II, Mg II) at maximum strength near A5; Ca II lines strengthening; lines of neutral metals appearing weakly</td>
</tr>
<tr>
<td>F</td>
<td>White stars; hydrogen lines declining; neutral metal lines increasing</td>
<td>Hydrogen lines weakening rapidly while H and K lines of Ca II strengthen; neutral metal (Fe I and Cr I) lines gaining on ionized metal lines by late F</td>
</tr>
<tr>
<td>G</td>
<td>Yellowish stars; many metal lines; Ca II lines dominate</td>
<td>Hydrogen lines very weak; Ca II H and K lines reach maximum strength near G2; neutral metal (Fe I, Mn I, Ca I) lines strengthening while ionized metal lines diminish; molecular G band of CH becomes strong</td>
</tr>
<tr>
<td>K</td>
<td>Reddish stars; molecular bands appear; neutral metal lines dominate</td>
<td>Hydrogen lines almost gone; Ca lines strong; neutral metal lines very prominent; molecular bands of TiO begin to appear by late K</td>
</tr>
<tr>
<td>M</td>
<td>Coolest reddish stars; neutral metal lines strong; molecular bands dominate</td>
<td>Neutral metal lines very strong; molecular bands prominent, with TiO bands dominating by M5; vanadium oxide bands appear</td>
</tr>
</tbody>
</table>
## Spectral Types

<table>
<thead>
<tr>
<th>Spectral Type</th>
<th>$M_V$</th>
<th>$B - V$</th>
<th>$T_{\text{eff}}$ (K)</th>
<th>BC</th>
<th>$R/R_{\odot}$</th>
<th>$M/M_{\odot}$</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>V</td>
<td>III</td>
<td>I</td>
<td></td>
<td>V</td>
<td>V</td>
</tr>
<tr>
<td>O5</td>
<td>-6.0</td>
<td>-0.32</td>
<td>-0.32</td>
<td>-0.32</td>
<td>50,000</td>
<td>-4.30</td>
</tr>
<tr>
<td>B0</td>
<td>-4.1</td>
<td>-5.0</td>
<td>-6.2</td>
<td>-0.30</td>
<td>27,000</td>
<td>-3.17</td>
</tr>
<tr>
<td>B5</td>
<td>-1.1</td>
<td>-2.2</td>
<td>-5.7</td>
<td>-0.16</td>
<td>16,000</td>
<td>-1.39</td>
</tr>
<tr>
<td>A0</td>
<td>+0.6</td>
<td>-0.6</td>
<td>-4.9</td>
<td>0.00</td>
<td>10,400</td>
<td>-0.40</td>
</tr>
<tr>
<td>A5</td>
<td>+2.1</td>
<td>+0.3</td>
<td>-4.5</td>
<td>+0.15</td>
<td>8200</td>
<td>-0.15</td>
</tr>
<tr>
<td>F0</td>
<td>+2.6</td>
<td>+0.6</td>
<td>-4.5</td>
<td>+0.30</td>
<td>7200</td>
<td>-0.08</td>
</tr>
<tr>
<td>F5</td>
<td>+3.4</td>
<td>+0.7</td>
<td>-4.5</td>
<td>+0.45</td>
<td>6700</td>
<td>-0.04</td>
</tr>
<tr>
<td>G0</td>
<td>+4.4</td>
<td>+0.6</td>
<td>-4.5</td>
<td>+0.60</td>
<td>6000</td>
<td>-0.06</td>
</tr>
<tr>
<td>G5</td>
<td>+5.2</td>
<td>+0.3</td>
<td>-4.5</td>
<td>+0.65</td>
<td>5500</td>
<td>-0.10</td>
</tr>
<tr>
<td>K0</td>
<td>+5.9</td>
<td>+0.2</td>
<td>-4.5</td>
<td>+0.81</td>
<td>5100</td>
<td>-0.19</td>
</tr>
<tr>
<td>K5</td>
<td>+8.0</td>
<td>-0.3</td>
<td>-4.5</td>
<td>+1.18</td>
<td>4300</td>
<td>-0.71</td>
</tr>
<tr>
<td>M0</td>
<td>+9.2</td>
<td>-0.4</td>
<td>-4.5</td>
<td>+1.39</td>
<td>3700</td>
<td>-1.20</td>
</tr>
<tr>
<td>M5</td>
<td>+12.3</td>
<td>-0.5</td>
<td>-4.5</td>
<td>+1.69</td>
<td>3000</td>
<td>-2.10</td>
</tr>
</tbody>
</table>

*All class Ia stars have an absolute visual magnitude of -0.7.

BC is bolometric correction.
Stellar Spectra and Stellar Atmospheres

Determined by (in decreasing order of importance)

1. $T_{\text{eff}}$
2. $g^*$
3. Composition
Spectral Luminosity Classes

\[ g_* = \frac{G M_*}{R_*^2} \]

\[ R_* \propto M_*^{0.75} \] for normal (main sequence) stars →

\[ g_* \text{ doesn’t vary too much} \]

Giants, supergiants → big \( R_* \)

White dwarfs → small \( R_* \)

\[ g_* \text{ mainly determined by } R_* \]

Fixed \( T_{\text{eff}} \), \( L = 4\pi R_*^2 \sigma T_{\text{eff}}^4 \) →

\( R_* \text{ changes } L \)

\( g_* \text{ gives } L \text{ or } R_* \)
## Spectral Luminosity Classes

<table>
<thead>
<tr>
<th>Class</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>Ia</td>
<td>Bright supergiants</td>
</tr>
<tr>
<td>Ib</td>
<td>Supergiants</td>
</tr>
<tr>
<td>II</td>
<td>Bright giants</td>
</tr>
<tr>
<td>III</td>
<td>Giants</td>
</tr>
<tr>
<td>IV</td>
<td>Subgiants</td>
</tr>
<tr>
<td>V</td>
<td>Main-sequence stars and dwarfs</td>
</tr>
</tbody>
</table>
Stellar Spectra and Stellar Atmospheres

Determined by (in decreasing order of importance)

1. $T_{\text{eff}}$
2. $g^*$
3. Composition
Stellar Composition

Mainly hydrogen and helium
## Solar Composition

<table>
<thead>
<tr>
<th>Element</th>
<th>Abundance by mass</th>
</tr>
</thead>
<tbody>
<tr>
<td>Hydrogen</td>
<td>73.5%</td>
</tr>
<tr>
<td>Helium</td>
<td>24.8%</td>
</tr>
<tr>
<td>Oxygen</td>
<td>0.788%</td>
</tr>
<tr>
<td>Carbon</td>
<td>0.326%</td>
</tr>
<tr>
<td>Nitrogen</td>
<td>0.118%</td>
</tr>
<tr>
<td>Iron</td>
<td>0.162%</td>
</tr>
</tbody>
</table>
Stellar Composition

Mainly hydrogen and helium

\[ X = \text{mass fraction of hydrogen} \approx 0.74 \ (90\% \ of \ atoms) \]
\[ Y = \text{mass fraction of helium} \approx 0.24 \ (10\% \ of \ atoms) \]
\[ Z = \text{mass fraction of heavier elements} \approx 0.02 \text{ in Sun} \ (0.1\% \ of \ atoms) \]
Stellar Composition

Mainly hydrogen and helium

- $X =$ mass fraction of hydrogen $\sim 0.74$ (90% of atoms)
- $Y =$ mass fraction of helium $\sim 0.24$ (10% of atoms)
- $Z =$ mass fraction of heavier elements $\sim 0.02$ in Sun (0.1% of atoms)

Fraction of heavy elements varies

- Population I = like Sun, $Z \sim 0.01$
- Population II = low abundances, $Z \sim 0.001$