Curve of Binding Energy

\[ E_{\text{bind}}/A \text{ (MeV)} \]

\[ A \text{ (# of nucleons)} \]

- \(^4\text{He}\)
- \(^{56}\text{Fe}\)
- \(^1\text{H}\)
Carbon Burning, Etc.

\[
\begin{align*}
^{12}\text{C} + ^4\text{He} & \rightarrow ^{16}\text{O} + \gamma \\
^{16}\text{O} + ^4\text{He} & \rightarrow ^{20}\text{Ne} + \gamma \\
^{20}\text{Ne} + ^4\text{He} & \rightarrow ^{24}\text{Mg} + \gamma \\
\cdots & \rightarrow ^{56}\text{Fe}
\end{align*}
\]

Explains why elements beyond Fe are very rare, can’t be made by exothermic fusion reactions.

Explains why most common elements beyond H have Z = N = even = multiples of He.
Carbon Burning, Etc.

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Explains why elements beyond Fe are very rare, can’t be made by exothermic fusion reactions.

Explains why most common elements beyond H have \( Z = N = \text{even} = \text{multiples of He} \).
relative abundance of the elements in the universe
Vertical: relative abundance, powers of ten
Horizontal: atomic number of element
Fusion Reactions

- **Sun**
- **CNO**
- **3α**
- **C burning**

<table>
<thead>
<tr>
<th>Temperature (K)</th>
<th>Log ε (erg/s/gm)</th>
</tr>
</thead>
<tbody>
<tr>
<td>7</td>
<td>7</td>
</tr>
<tr>
<td>7.5</td>
<td>7.5</td>
</tr>
<tr>
<td>8</td>
<td>-2</td>
</tr>
</tbody>
</table>

Diagram showing the relationship between temperature (log T in K) and the logarithm of the energy production rate (log ε in erg/s/gm). The diagram highlights the different fusion reactions at various temperatures, including the Sun, CNO, 3α, and C burning processes.
Thermal Stability of Stars

Fusion:
- Makes lots of energy
- Very temperature sensitive

\{ \text{explosives} \}

BANG
Thermal Stability of Stars

Why don’t stars explode?
Stars have negative heat capacity!
Energies is Stars

Stars have **negative heat capacity**

If you add energy (heat), they get cooler!!

\[
\langle PE \rangle = -2\langle TE \rangle
\]

\[
E = -\langle TE \rangle
\]
Thermal Stability of Stars

Why don’t stars explode?
Stars have negative heat capacity!
Add energy, get cooler
Add energy, pressure increases, gas expands, gas gets cooler
Cooler $\Rightarrow$ less fusion $\Rightarrow$ less energy
Fusion in stars $\Rightarrow$ perfect thermostat!
Stars are incredibly stable!
Thermal Stability of Stars

Exceptions:

1. Virial theorem applies to whole of star
   Parts of star exchange energy $\rightarrow$ pulsations

2. Thermal gas pressure only
   $KE = -E$
   $KE = (3/2) PV = (3/2) NkT$ only if $P = P_{\text{gas}}$
   Degeneracy pressure, $KE = (3/2) P_{\text{deg}} V$, but
   $P_{\text{deg}} = \text{func. of density only, not temperature}$

Fusion in degenerate regions $\rightarrow$ explosion
Fusion – The Key to the Stars
Fusion – The Key to the Stars

- Make stellar energy, provide source of light
- Make all heavy elements (C, N, O, Fe, …)
- Make stars stable (or unstable)
- Change composition of stars → they evolve
- Set lifetime of stars, \( t = \frac{E_{\text{nucl}}}{L} \)
Tests of Stellar Structure Theory
ASTR 2120

Davis Solar Neutrino Experiment
Tests of Stellar Structure Theory

At some level, all of astronomy (rest of this course)
Critical tests?
Sun is good since it is close
Solar Oscillations

Helio-seismology

$P \sim (G \langle \rho \rangle)^{-1/2}$

But, many normal modes of oscillation
Can also be done for other stars
Solar Oscillations

One normal made of Sun (red and blue shifts)
Solar Oscillations

Some modes probe surface, some deep interior
Solar Oscillations

Spectrum of frequencies of oscillation
Solar Neutrino Experiments

Test idea: Fusion powers stars
Cannot see into Solar core with light, but neutrinos should escape
Unfortunately, main neutrinos made by pp reaction are very low energy, hard to detect
Detect side reactions
Solar Neutrino Experiments

The diagram illustrates the pp chain, which is a series of nuclear reactions in the Sun that produce solar neutrinos. The reactions involve protons and neutrons, and the process is initiated by the proton-proton cycle (pp process) where two protons combine to form a deuteron (2H) and a positron (e+) and a neutrino (v_e).

1. \( p^+ + p^+ \rightarrow 2H + e^+ + v_e \) with 99.77% probability.
2. \( \frac{3}{2}H + p^+ \rightarrow \frac{3}{2}He + \gamma \) with 15.08% probability.
3. \( \frac{3}{2}He + \frac{3}{2}He \rightarrow \frac{4}{2}He + 2p^+ \) with 84.92% probability.
4. \( \frac{3}{2}Li + p^+ \rightarrow \frac{4}{2}He + \frac{4}{2}He \) with 0.23% probability.
5. \( \frac{3}{2}He + e^- \rightarrow \frac{7}{2}Be \) with 99.9% probability.
6. \( \frac{7}{2}Be + e^- \rightarrow \frac{7}{2}Li + v_e \) with 0.1% probability.
7. \( \frac{7}{2}Be + p^+ \rightarrow \frac{8}{2}Be + \gamma \) with 0.23% probability.
8. \( \frac{8}{2}Be \rightarrow \frac{8}{2}Be + e^+ + v_e \) with 0.1% probability.

The diagram also shows the hep (helium-3) cycle and the ppIII cycle, which are other processes that contribute to the production of solar neutrinos.

The energy released in these processes is 14 Mev!
Solar Neutrino Experiments

Bahcall–Serenelli 2005

Neutrino Spectrum (±1σ)

Flux (cm⁻² s⁻¹)

Neutrino Energy in MeV

pp ± 1%

7Be ± 10.5%

17N ± 10.5%

14N

pep ± 2%

17P ± 10.5%

8B + ± 16%

hep ± 16%
Solar Neutrino Experiments
Davis Experiment

Mainly detect $^8\text{B} \ \nu$’s
$\nu + ^{37}\text{Cl} \rightarrow ^{37}\text{Ar} + e^-$
Normal chlorine - use 100,000 gal of $\text{C}_2\text{Cl}_4$ (dry-cleaning fluid)
Argon inert, bubble out of tank
$^{37}\text{Ar}$ decays (beta decay), detect every single atom!
Homestead Gold Mine, South Dakota
Davis Experiment
Davis Experiment
Davis Experiment

Predict ~ 5.6 SNU (solar neutrino units)
Observe only \(1.8 \pm 0.3\) SNUs ~ 1/3 expected

What is wrong?

1. Experiment wrong - No
2. Nuclear reaction rates wrong - No
3. Astronomy wrong (solar model incorrect) - No
4. Physics wrong - Yes!
   Neutrinos are NOT mass-less and stable
Neutrino Oscillations

Three flavors of neutrinos

$\nu_e$  

$\nu_\mu$  $\nu_\tau$
Neutrino Oscillations

electron–neutrino → solar material → muon–neutrino
Sudbury Experiment
Super-Kamiokande Experiment
Neutrino Oscillations

Three flavors of neutrinos

\[ \nu_e \quad \nu_{\mu} \quad \nu_\tau \]

Yes!
The Nobel Prize in Physics 2002

"for pioneering contributions to astrophysics, in particular for the detection of cosmic neutrinos"
Five Equations

\[
\begin{align*}
\frac{dP}{dr} &= -\frac{GM(r)\rho}{r^2} \quad \text{Hydrostatic equilibrium} \\
\frac{dM}{dr} &= 4\pi r^2 \rho \quad \text{Mass} \\
\frac{dL}{dr} &= 4\pi r^2 \varepsilon \rho \quad \text{Thermal equilibrium} \\
\frac{dT}{dr} &= -\min \left\{ \frac{3k\rho}{64\pi \sigma r^2} \frac{1}{T^3} L(r) \quad \text{radiation}, \right. \\
&\quad \left. -\frac{2T}{5} \frac{dP}{dr} \quad \text{convection} \right\} \\
\frac{dL}{dP} &= -\frac{\rho kT}{\mu m_p} + P_{\text{rad}} + P_{\text{deg}} \quad \text{Equation of state}
\end{align*}
\]
Boundary Conditions

Take radius $r$ as independent variable

\[ P(r), T(r), \rho(r), L(r), M(r) \quad 0 \leq r \leq R_* \]

Boundary Conditions:

Center ($r = 0$): \[ M(0) = 0, \quad L(0) = 0 \]

Surface ($r = R_*$): \[ M(R_*) = M_*, \quad P(R_*) = 0, \quad \rho(R_*) = 0 \]

Two Problems:

- 4 differential equations, 5 boundary conditions

What is $R_*$?
Boundary Conditions

Change independent variable to mass $M$

$r(M), P(M), T(M), \rho(M), L(M), \quad 0 \leq M \leq M_*$

$$\frac{dM}{dr} = 4\pi r^2 \rho \quad \Rightarrow \quad \frac{dr}{dM} = \frac{1}{4\pi r^2 \rho}$$

$$\frac{dX}{dM} = \frac{dX}{dr} \frac{dr}{dM}$$
Five (New) Equations

\[ \frac{dr}{dM} = \frac{1}{4\pi r^2 \rho} \quad \text{radius (formerly mass)} \]

\[ \frac{dP}{dM} = -\frac{GM(r)}{4\pi r^4} \quad \text{Hydrostatic equilibrium} \]

\[ \frac{dL}{dM} = \varepsilon \quad \text{Thermal equilibrium} \]

\[ \frac{dT}{dM} = -\min \left\{ \frac{3\kappa}{256\pi^2 \sigma r^4} \frac{1}{T^3} L(r) \right\} \quad \text{radiation} \]

\[ \frac{2}{5} \frac{T}{P} \frac{dP}{dM} \quad \text{convection} \]

\[ P = \frac{\rho k T}{\mu m_p} + P_{rad} + P_{\text{deg}} \quad \text{Equation of state} \]
Boundary Conditions

Center ($M = 0$): $r(0) = 0$, $L(0) = 0$

Surface ($M = M_*$): $P(M_*) = 0$, $\rho(M_*) = 0$

Better!

4 differential equations $\iff$ 4 boundary conditions

$R_*$ is derived
Vogt-Russell Theorem

Necessary inputs:

- Mass of star $M_*$
- Mean particle mass $\mu$, opacity $\kappa$, energy production $\varepsilon$
  Depend on $\rho$, $T$, composition

Vogt-Russell Theorem:

Star is uniquely determined by mass $M_*$ & composition
Solar Model

\[ \rho_c = 147 \text{ gm/cm}^3 \]
\[ T_c = 15 \text{ million K} \]

Burning hydrogen in core \( \sim \) 20% of radius

Radiative zone \( \sim \) inner 85% of radius
Convective zone \( \sim \) outer 15% of radius
Main Sequence

Choice of composition:

Try mainly hydrogen and helium
  Surface of Sun and most other stars
  Interstellar gas \( \rightarrow \) what stars form from

Vary mass \( M_* \)

Reproduce main sequence (normal stars)

All burning H in their cores

Main Sequence = stars made of hydrogen burning H in their cores
Main Sequence

- **High mass**
- **Low mass**
Main Sequence

Hydrogen:
- Most abundant element
- Best nuclear fuel
- Burned first

Stars spend most of life on main sequence, only leave it when they start to die
# Sub-Stellar Objects

<table>
<thead>
<tr>
<th>Mass</th>
<th>Stars</th>
<th>Brown Dwarfs</th>
<th>Planets</th>
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<tbody>
<tr>
<td>$M \geq 0.075 , M_\odot$</td>
<td>$M \geq 0.012 , M_\odot$</td>
<td>$0.012 , M_\odot \leq M \leq 0.075 , M_\odot$</td>
<td>$M \leq 0.012 , M_\odot$</td>
</tr>
<tr>
<td>$\geq 78 , M_J$</td>
<td>$13 , M_J \leq M \leq 78 , M_J$</td>
<td>$\leq 13 , M_J$</td>
<td></td>
</tr>
<tr>
<td>Fusion?</td>
<td>Burn H</td>
<td>Burn D = $^2$H deuterium</td>
<td>No fusion</td>
</tr>
<tr>
<td></td>
<td>Later He, C, …</td>
<td>Short time, then cool</td>
<td></td>
</tr>
<tr>
<td></td>
<td></td>
<td>Degeneracy pressure</td>
<td></td>
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</tbody>
</table>

Star $M \geq 100 \, M_\odot$ ➔ blown apart by radiation pressure while forming?
Sub-Stellar Objects

Define “star” as self-gravitating object which can fuse $^1$H (normal hydrogen)

$M > 0.075 \, M_\odot$

Define “planet” as self-gravitating object which never has any fusion?

$M < 0.012 \, M_\odot = 13 \, M_J$

Must planets be formed orbiting stars?

Brown dwarfs intermediate
Brown Dwarfs

Never hot enough to fuse $^1\text{H}$ (normal hydrogen)

Hot enough to fuse $^2\text{H}$ (deuterium), but

$^2\text{H}$ (deuterium) rare ($2 \times 10^{-5}$ than of $^1\text{H}$)

Makes less energy

Burn deuterium for a short time, then cool down

Supported by electron degeneracy pressure

Like white dwarfs

Predicted in early 1960’s, discovered in 1994
Brown Dwarfs

Brown Dwarf Gliese 229B

Palomar Observatory
Discovery Image
October 27, 1994

Hubble Space Telescope
Wide Field Planetary Camera 2
November 17, 1995

PRC95-48 - ST ScI OPO - November 29, 1995
T. Nakajima and S. Kulkarni (CalTech), S. Durrance and D. Golimowski (JHU), NASA
Sub-Stellar Objects

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         | $13 \, M_{J} \leq M \leq 78 \, M_{J}$ | $M \leq 0.012 \, M_{\odot}$
         | $\leq 13 \, M_{J}$          |
| Fusion?| Burn H
         | Later He, C, …            | Burn D = $^2$H deuterium
         |                                    | Short time, then cool
         |                                    | Degeneracy pressure                | No fusion |

Star $M \geq 100 \, M_{\odot}$ ➔ blown apart by radiation pressure while forming?