Black Holes
ASTR 2110
Sarazin

Calculation of Curved Spacetime near Merging Black Holes
Test #2

Monday, November 17, 11 - 11:50 am
Chem 305 (classroom)
Bring pencils, paper, calculator

~2/3 Quantitative Problems (like homework problems)

~1/3 Qualitative Questions
Multiple Choice, Short Answer, Fill In the Blank questions
No essay questions
Test #1 (Cont.)

No problem set week of November 11 – 17 to allow study for test

Review Session:
Discussion session
Friday, November 14, 3-4 pm
Black Holes
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Calculation of Curved Spacetime near Merging Black Holes
Black Holes

Gravity crushes star to a point = singularity
Surrounded by "surface" from which nothing can escape = event horizon
Schwarzschild Metric for Black Hole

Karl Schwarzschild (1916):
Spherical coordinates. Non-rotating black hole

\[
(ds)^2 = \left( c \sqrt{1 - \frac{2GM}{c^2 r}} \, dt \right)^2 - \left( \frac{dr}{\sqrt{1 - \frac{2GM}{c^2 r}}} \right)^2 - (r \, d\theta)^2 - (r \sin \theta \, d\phi)^2
\]

\[
(ds)^2 = \left( c \sqrt{1 - \frac{R_s}{r}} \, dt \right)^2 - \left( \frac{dr}{\sqrt{1 - \frac{R_s}{r}}} \right)^2 - (r \, d\theta)^2 - (r \sin \theta \, d\phi)^2
\]

\[R_s \equiv \frac{2GM}{c^2} = 3 \text{ km} \left( \frac{M}{M_\odot} \right)\]
Schwarzschild Metric

2 odd places

\[ r \to 0, \quad \frac{2GM}{c^2 r} \to \infty \]  \hspace{1cm} \text{singularity}

\[ r = R_s, \quad (dt)^2 \text{ term } \to 0, \quad (dr)^2 \text{ term } \to \infty \]  \hspace{1cm} \text{event horizon}

\[ (ds)^2 = \left(c \sqrt{1 - \frac{2GM}{c^2 r}} \ dt\right)^2 - \left(\sqrt{1 - \frac{2GM}{c^2 r}} \ dr\right)^2 - (r \ d\theta)^2 - (r \sin \theta \ d\phi)^2 \]
Schwarzschild Metric

event horizon ≠ physical singularity, just coordinate singularity

Like North Pole on Earth
Black Hole Physics

Singularity = Doom

Tidal forces $\Rightarrow \infty$

Worry:

$\infty$ Physics goes to pot!?
Black Hole Physics

Nothing Escapes Event Horizon

\[
(ds)^2 = \left(c \sqrt{1 - \frac{R_s}{r}} \ dt\right)^2 - \left(\frac{dr}{\sqrt{1 - \frac{R_s}{r}}}\right)^2 - (r \ d\theta)^2 - (r \sin \theta \ d\phi)^2
\]

\(r < R_s \Rightarrow \frac{R_s}{r} > 1 \Rightarrow \sqrt{1 - \frac{R_s}{r}}\) imaginary, \(\left(\sqrt{1 - \frac{R_s}{r}}\right)^2 < 0\)

\((dt)^2\) term negative (normally positive)

\((dr)^2\) term positive (normally negative)

\(r \leftrightarrow t\) radius and time reverse roles!!
Black Hole Physics

Nothing Escapes Event Horizon

"Now, here, you see, it takes all the running you can do, to keep in the same place. If you want to get somewhere else, you must run at least twice as fast as that!"

-- The Red Queen
Black Hole Physics

Cosmic Censorship?

There are no "naked singularities", uncovered by a horizon.

XXX Rated

"The Naked Singularity"

Sexual acts performed

Really Hot" - Penthouse
Black Hole Physics

Nothing Really Happens @ Event Horizon

Infalling person:
- falls through horizon, nothing happens

Outside observer:
- first speed up, then slow down, redshift, fade, hang forever just above
Black Hole Physics

Nothing Really Happens @ Event Horizon

Infalling person:
falls through horizon, nothing happens
Black Hole Physics

Nothing Really Happens @ Event Horizon

Infalling person:
falls through horizon, nothing happens

Outside observer:
first speed up, then slow down, redshift, fade, hang forever just above
Infall into Black Hole

infalling person

outside observer

infall slows, $v \rightarrow 0$, $\infty$ redshift, fades away, hovers forever just above horizon
Black Hole Physics

Orbits near Black Hole

In general, complex. Consider circular orbits

Newton: any radius possible

GR: No stable circular orbits $r \leq 3 R_S$
“Black Holes Have No Hair”

No matter what they swallow, only retain:

Mass $M$, Charge $Q$, Spin Angular Momentum $J$

Interstellar matter, very high electrical conductivity

$\rightarrow$ “shorts out” charge

Astrophysical Black Holes:

Mass $M$, Spin Angular Momentum $J$
Spinning Black Holes

Kerr Metric (Boyer-Linquist Coordinates)

\[
(ds)^2 = \left(1 - \frac{R_s r}{\rho^2}\right)(c \, dt)^2 - \frac{\rho^2}{\Delta} dr^2 - \left(\rho \, d\theta\right)^2
- \left(r^2 + \alpha^2 + \frac{R_s r \alpha^2}{\rho^2} \sin^2 \theta\right)(\sin \theta \, d\phi)^2 + \frac{2R_s r \alpha \sin^2 \theta}{\rho^2} c \, dt \, d\phi
\]

\[R_s = \frac{2GM}{c^2}\] Schwarzschild radius

\[\alpha \equiv \frac{J}{Mc}\] (units of length)

\[\rho^2 \equiv r^2 + \alpha^2 \cos^2 \theta\] (units of length)

\[\Delta \equiv r^2 - R_s r + \alpha^2\] (units of length^2)
Spinning Black Holes

Kerr Metric

Exterior Solution

Event Horizon

Oblate spheroid “ergosphere”

Can escape, but must rotate with black hole

top view

side view
Spinning Black Holes

Kerr Metric

Interior Solution
Ring singularity
Inner Horizon!

But interior solution is unstable
Spinning Black Holes

Kerr Metric

Worm holes??

Unstable $\rightarrow$ closes as soon as anything enters it!

white hole

black hole

Closed wormhole
Energy from Black Holes

Penrose Mechanism

Rotating BH

Throw something out backwards

\[ E_{\text{out}} \approx E_{\text{in}} + \Delta mc^2 \]

May not be important in nature (too complicated), but …
Solution to Mankind’s Problems

Biggest problems:

- Get rid of garbage, pollution
- Make energy
- Build city around rotating BH
Energy from Black Holes

Natural Method

Drop matter into BH carelessly
Bump into, rub against other matter
Friction $\rightarrow$ KE $\rightarrow$ heat $\rightarrow$ light
Virial Thm.
Heat $\approx (1/2) |PE| \approx (1/2) \frac{GMm}{R_S}$
$\approx (1/2) \frac{GMm}{(2GM/c^2)}$

$E_{out} = \text{Heat} \approx mc^2/4$ !!!

Accretion by BHs is biggest important energy source in Universe
Black Hole Physics

Black Hole Surface Areas

When BHs merge, mass can increase or decrease
Total surface area of BH horizons always increases in GR

2\textsuperscript{nd} Law of BH Dynamics
Entropy always always increases

\begin{align*}
S &= \frac{c^3 k}{4\hbar G} A \quad \text{Entropy} \\
T &= \frac{\hbar c^3}{8\pi kGM} \quad \text{Temperature}
\end{align*}
The Death of Stars
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Cat’s Eye Planetary Nebula
Low Mass Star ($1 \, M_{\odot}$)

Left as AGB (red supergiant) with C/O core, giant H envelope

H envelope, ~ few AU

C/O core, ~ $10^9$ cm, degenerate & inert
Low Mass Star - Death

Asymptotic Giant Branch

Red supergiant

Inert carbon/oxygen core
Low Mass Star (1 $M_\odot$) - Cont.

Inert core, must die

Envelope very extended, cool.

Hydrogen recombines: $H^+ + e^- \rightarrow H + 13.6$ eV

Gravitational binding energy per atom =

$$G M m_H / R \approx 10 \text{ ev} (M / M_\odot)(R/\text{AU})^{-1}$$

H shell ejected, $(1/2) m_H v^2 \sim \text{ few eV}$,

$$v \sim 20 \text{ km/s}$$

Planetary Nebula
Low Mass Star - Death
Planetary Nebula & White Dwarf

Degenerate core revealed, becomes a white dwarf

C/O for Sun

other composition for other masses

Core initially very hot, $T_{\text{eff}} \sim 100,000$ K, lots of UV, ionizes planetary nebula

Fast wind from WD sweeps envelope into shell
Low Mass Star - Death

The image is a Hertzsprung-Russell (H-R) diagram illustrating the evolution of low mass stars. The diagram shows the relationship between surface temperature and luminosity, which are key parameters in understanding stellar evolution. The diagram includes various stages of a star's life, from the main sequence (MS) to the eventual formation of a white dwarf. Points A through G represent different stages along the evolutionary path, with labels indicating specific conditions such as Planetary nebula ejected and Asymptotic giant branch.
Planetary Nebula & White Dwarf

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Planetary Nebula
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Hourglass Nebula · MyCn18
HST · WFPC2
PRC96-07 · ST ScI OPO · January 16, 1996
R. Sahai and J. Trauger (JPL), the WFPC2 Science Team and NASA
Planetary Nebula
Planetary Nebula & White Dwarf

Degenerate core revealed, becomes a white dwarf

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Core initially very hot, $T_{\text{eff}} \sim 100,000$ K, lots of UV, ionizes planetary nebula

Fast wind from WD sweeps envelope into shell

WD cools, fades away
Low Mass Star - Death

[Diagram showing the lifecycle of a low mass star, with labels for different stages and spectral classifications.]