Mie Theory for Optical Properties of Dust Grains
Assumes spherical, homogeneous, isotropic grains and classical EM interactions

Definitions:

\[ a \equiv \text{radius of grain} \]
\[ \lambda \equiv \text{wavelength of light} \]
\[ m \equiv \text{complex index of refraction} \]

Characterize grain by

\[ x \equiv \frac{2\pi a}{\lambda} \]
\[ y \equiv mx \]

EM wave solution in terms of spherical Bessel functions \( j_n(x) \) and \( h_n^{(2)}(x) \):

\[ \psi_n(x) \equiv x j_n(x) \]
\[ \zeta_n(x) \equiv x h_n^{(2)}(x) \]

Scattering amplitudes: \( a_n \) for incident wave with polarization \( E \perp \) scattering plane; \( b_n \) for incident polarization \( E \parallel \) scattering plane

\[ a_n = \frac{\psi_n'(y) \psi_n(x) - m \psi_n(y) \psi_n'(x)}{\psi_n'(y) \zeta_n(x) - m \psi_n(y) \zeta_n'(x)} \]
\[ b_n = \frac{m \psi_n'(y) \psi_n(x) - \psi_n(y) \psi_n'(x)}{m \psi_n'(y) \zeta_n(x) - \psi_n(y) \zeta_n'(x)} \]

Efficiencies for extinction \( (Q_e) \), scattering \( (Q_s) \), and absorption \( (Q_a) \) are:

\[ Q_e = \frac{2}{x^2} \text{Re} \left[ \sum_{n=1}^{\infty} (2n + 1)(a_n + b_n) \right] \]
\[ Q_s = \frac{2}{x^2} \sum_{n=1}^{\infty} (2n + 1) \left( |a_n|^2 + |b_n|^2 \right) \]
\[ Q_a = Q_e - Q_s \]