Summary of Coulomb Collisions

Let $e$ denote electrons, $i$ denote general positive ions, and $p$ denote protons. Unless otherwise noted, assume that the ions are protons, that $n_e \approx n_p$, and that the electrons and protons have MB thermal distributions with $T_e \approx T_i = T_p$. A wide range of distance scales contribute to Coulomb collisions, with the result that the rate of Coulomb collisions depends on the Coulomb logarithm:

$$\ln \Lambda \equiv \ln \left( \frac{b_{\text{max}}}{b_{\text{min}}} \right) \approx 30 + \ln \left[ \frac{T}{10^6 \text{ K}} \left( \frac{n_e}{1 \text{ cm}^{-3}} \right)^{-1/2} \right].$$

Here, $b_{\text{max}}$ and $b_{\text{min}}$ are the maximum and minimum impact parameters contributing to the collisions. This assumes the quantum limit which applies at high velocity ($v \gtrsim \alpha Zc \sim 0.01 Zc$).

Then, the slowing down time for electron colliding with electrons is

$$\tau_s(e,e) = \frac{3 \sqrt{\pi} m_e^2}{16 \pi e^4 n_e \ln \Lambda} \left( \frac{2kT_e}{m_e} \right)^{3/2} \approx \frac{0.28 T_e^{3/2}}{n_e \ln \Lambda} \text{ sec (cgs units)}.$$

The timescale $\tau_s(e,p)$ for electron to collide with ions (but not to share energy) is similar. The timescale for ions of charge $Z_i e$ and mass $A_i m_p$ to collide with one another is

$$\tau_s(i,i) = \frac{n_e}{n_i} \sqrt{\frac{m_i}{m_e}} \left( \frac{T_i}{T_e} \right)^{3/2} \frac{1}{Z_i^2} \tau_s(e,e) \approx \frac{11.4 A_i^{1/2} T_i^{3/2}}{n_e Z_i^4 \ln \Lambda} \text{ sec (cgs units)}.$$

The timescale for electrons and ions to collide and share energy is

$$\tau_s(i,e) = \frac{m_i}{m_e} \frac{1}{Z_i^2} \tau_s(e,e) \approx \frac{500 A_i T_e^{3/2}}{n_e Z_i^2 \ln \Lambda} \text{ sec (cgs units)}.$$

If the gas is mainly hydrogen with $i = p$, $n_e \approx n_p$, and $T_e \approx T_i = T_p$, then

$$\tau_s(p,p) \approx \sqrt{\frac{m_p}{m_e}} \tau_s(e,e) \approx 43 \tau_s(e,e)$$

$$\tau_s(p,e) \approx \sqrt{\frac{m_p}{m_e}} \tau_s(p,p) \approx 43 \tau_s(p,p) \approx \frac{m_p}{m_e} \tau_s(e,e) \approx 1800 \tau_s(e,e).$$

The result of these different timescales is that: First, electrons $\rightarrow$ isotropic MB distribution at $T_e$ on timescale $\tau_s(e,e)$. Then, ions $\rightarrow$ isotropic MB distribution at $T_i$ on timescale $\tau_s(i,i)$. Then, electrons and ions $\rightarrow$ equipartition with $T_e = T_i$ on timescale $(1/2)\tau_s(i,e)$.

**Electrical Conductivity:** Mainly due to electrons

$$\bar{J} = \sigma \bar{E} \quad \sigma \approx \frac{3m_e}{e^2 \ln \Lambda} \left( \frac{2kT_e}{\pi m_e} \right)^{3/2}.$$

**Thermal Conductivity:** Mainly due to electrons

$$\bar{Q} = -\kappa \bar{\nabla} T \quad \kappa \approx \frac{7m_e^2 k}{e^4 \ln \Lambda} \left( \frac{2kT_e}{\pi m_e} \right)^{5/2}.$$

**Dynamic Viscosity:** Mainly due to ions

$$\eta \approx \frac{2 \sqrt{m_p}}{5 e^4 \ln \Lambda} (kT_p)^{5/2}.$$