

# Selection of Homework Questions

[Next](#)[Prev](#)[Print](#)

## Topic 1: History & Preliminaries

### (1) Quick estimates using "psm" units (see : Topic 1.3e [< link >](#) )

- a. A globular cluster moves at 150 km/s on a circular orbit of radius 25 kpc.
  - (i) What's the period of the cluster?
  - (ii) What's the mass and mean density interior to its orbit?
  - (iii) If the cluster has size  $\sim 10$  pc and internal velocity dispersion  $\sim 10$  km/s, what's its mass and mean density?
- b. What, roughly, are the speeds and periods of :
  - (i) comets in the Oort cloud at a radius of  $\sim 1/4$  pc.
  - (ii) gas in an accretion disk 1000 AU from a  $10^8 M_{\odot}$  black hole.
  - (iii) stars orbiting in the galactic center cluster of size  $\sim 30$  arcsec and density  $\sim 10^6 M_{\odot} \text{pc}^{-3}$
- c. Cosmological estimates are often easy using psm units:
  - (i) Hubble's original estimate was  $H_0 = 530 \text{ km s}^{-1} \text{Mpc}^{-1}$ . What cosmic age does this imply?
  - (ii) For  $H_0 = 72 \text{ km s}^{-1} \text{Mpc}^{-1}$ , what's the critical density in  $M_{\odot} \text{pc}^{-3}$  (use  $\rho_c = 3H_0^2/8\pi G$ ).
  - (iii) Compare this to the density of typical clusters of size  $\sim 1$  Mpc and dispersion  $\sim 500 \text{ km s}^{-1}$ .
  - (iv) If such clusters virialize by dissipationless collapse (i.e.  $R_{\text{final}} \sim 1/2 R_{\text{initial}}$ ), what was their density just before collapse? If collapse starts when  $\delta\rho/\rho \sim 1$  (ie they have twice the mean density), what redshift did the collapse begin?
- d. Consider the formation of ellipticals:
  - (i) What's the total kinetic energy (KE) of a giant E galaxy with  $L \sim 10^{11} L_{\odot}$ ,  $\sigma \sim 300 \text{ km s}^{-1}$  and  $M/L \sim 10$ ?
  - (ii) From the virial theorem, the galaxy's binding energy (BE) is equal to its KE, which was liberated when it formed. If it collapsed on its current dynamical timescale, what was the "collapse luminosity",  $L_{\text{BE}}$ , in PLU and  $L_{\odot}$ ?
  - (iii) Is this significant compared to the typical luminosity of the associated starburst?

### (2) Magnitudes and Mass to Light Ratios :

- a. If  $I_B$  is the B band surface luminosity density measured in  $L_{\odot} \text{pc}^{-2}$  and  $\mu_B$  is the corresponding B band surface brightness in B mag arcsec $^{-2}$  (mag/ss), show that

$$\mu_B = 27.05 - 2.5 \log(I_B)$$

Derive similar relations for V, I, and bolometric (you will need to use the solar absolute magnitudes given in the notes : [< link >](#) ). These are potentially very useful relations. (B&M Q2.2)

- b. NGC 1399 has a central surface brightness of  $\mu_V \sim 16.0$  mag/ss. What is its projected surface luminosity density,  $I_B$ , in  $L_{\odot, V} \text{pc}^{-2}$  ?
- c. If it has a core radius of  $\sim 1.0$  arcsec, a redshift of 1350 km/s, and a central velocity dispersion of  $\sim 150$  km/s, estimate the core's luminosity density and mass density, and hence its M/L $_V$  ratio. Include "h" in your answers.

### (3) You outshine the stars !

Putting mass-to-light ratios into physical units can be surprising :

- a. What's M/L $_{\text{bol}}$  for the sun, in units of kg/Watt? About 50% of the sun's mass is in its nuclear burning core. What's the M/L $_{\text{bol}}$  of this "nuclear furnace"? Think about this value: does it jive with the cliché of the sun as a "roaring fusion furnace"? You should conclude that stars are **not** luminous because they're intrinsically luminous **per kg** -- indeed they are quite feeble in that sense. They are luminous only because their furnaces are so **massive**.
- b. To emphasise this, estimate **your own** M/L $_{\text{bol}}$  ratio, using the same units. Assume you weigh 100 kg and radiate like a black body of area 2 m $^2$  at 300K -- not unreasonable.
- c. Imagine a big ball of  $2 \times 10^{28}$  people -- a bizarre and fairly unpleasant notion -- it has about the same mass and size as the sun. Assume that people survive until they starve (they don't eat each other!) so that the "lifetime" of this human star is about 1 week.

(i) What would its luminosity be?

(ii) Compare the **total** energy liberated by the sun and the human star integrated over their respective lifetimes.

(iii) Compare this ratio to the ratio of typical electron binding energies (chemistry) with typical nuclear binding energies (fusion).

Notice: solar type stars live so long for **two** reasons: (a) their fuel is indeed very energy rich; but (b) they burn it at an extremely frugal rate -- their furnaces are surprisingly feeble, per kg, well below even to your own metabolic rate.

- d. Finally, use the stellar mass-luminosity relation ( $L \propto M^{3.5}$ ) to find out what star mass and spectral type has roughly the same M/L $_{\text{bol}}$  as you.

When your supervisor next asks you whether you have "fire in the belly" for your work, you can honestly reply, "more, even, than the sun and stars!"

#### (4) Alien Astronomers in Virgo study the Milky Way

In Topic 1.3b we learned that the Milky Way had an exponential disk scale length of 3.5 kpc with surface brightness near the sun (8.5 kpc) of  $15 L_{V,\odot} \text{pc}^{-2}$ , and a total luminosity of  $L_V = 1.4 \times 10^{10} L_{V,\odot}$ . Estimate  $m_V$  &  $M_V$  for the MW as viewed from the Virgo cluster (distance 15 Mpc). What is the surface brightness of the disk ( $\mu_V$  in magnitudes per square arcsec) at the radius of the sun and at the center; and what is the diameter, in arcsec, as defined by the 25<sup>th</sup> V mag/arcsec isophote. Assuming the rotation curve stays flat at  $\sim 220$  km/s outside the solar circle, what is the value of  $M/L_V$  measured inside the  $D_{25}$  isophote?

#### (5) Concordance Cosmology :

Use the parameter diagrams given in the notes, 1.3m ( [link](#) ) to gain familiarity with some of the basic properties of the concordance model.

- "More distant objects look smaller" Not true. Out to what redshift does this statement hold true ? Over what range of redshift does a 10 kpc galaxy appear to be between 1 and 1.5 arcsec in size ? What is the % error in using the small angle formula and linear Hubble law in estimating angular sizes at  $z \sim 0.1$ ; 0.5; 1 ?
- At what redshift was the expansion of the universe "coasting" (ie neither decelerating nor accelerating) ? At what fraction of the current age did this occur ?
- What is the look-back time to a  $z \sim 7$  quasar ? How long was it since the big bang, in Gyr and as a fraction of the current age ? How much denser was the universe at that time, and what was the CMB temperature ?
- JWST will see galaxies at  $z \sim 10$ . If their diameters are  $\sim 20$  kpc, how big will they appear to be ? Pushing further, LOFAR expects to see HI (21 cm) emission at  $z \sim 30$  during the "Dark Age" (after recombination but before reionization). How big will 100 kpc structures appear at that time ? Yet further, at recombination ( $z \sim 1000$ ) what physical scale corresponds to 1 degree (the first acoustic peak in the CMB power spectrum).
- Recombination occurs at  $z \sim 1000$ . Roughly, what's the hydrogen density at this time (in  $\text{cm}^{-3}$ ; allowing for the fact that baryons are only  $\sim 10\%$  of the total matter density). What's the ratio of radiation density to hydrogen density at this time ? At what redshift and corresponding cosmic age is the total matter density equal to the radiation density ? What's the temperature at that time ?

#### (6) Scaled $H_0$ :

A few examples of dealing with different values of  $h = H_0 / 100$ .

- What are the  $h$  dependencies of : diameter (from angular size); mass density (from dynamics);

luminosity (from flux); mass (from dynamics); and M/L.

- b. An old paper by Sandage uses  $H_0=55$  km/s/Mpc to give the central luminosity density and mass density in a galaxy to be  $10^3 L_{\odot}/\text{pc}^3$  and  $10^4 M_{\odot}/\text{pc}^3$ . Express these values using "h" and evaluate them for  $H_0=72$  km/s/Mpc.

---

[Home](#)

[Main](#)

[Index](#)

[Links](#)