Selection of Homework Questions

Topic 5: Spirals

(1) Bulge-Disk Surface Brightness Profiles :

An S0 galaxy has a spherical de Vaucouleurs $R^{1/4}$ law bulge with effective radius R_e = 0.5 arcmin, and Σ_e = 20.5 mag/ss. Its disk has **observed** (extrapolated) central surface brightness $\Sigma(0)$ = 20.8 mag/ss with major axis scale length R_d = 1.15 arcmin. The disk is inclined by 60 degrees to the plane of the sky.

Plot the observed major and minor axis surface brightness profiles for the bulge, disk, and total light (surface brightness in mag/ss *vs* angular scale in arcminutes). For the major axis, show where light is contributed by the bulge and disk in the ratios 1:5, 1:1, 5:1. For both plots, show a typical B-band surface brightness for a dark night sky, and the "limiting radius" corresponding to a surface brightness Σ = 26.5 mag/ss. What is the integrated bulge/disk luminosity ratio?

(hint : recall that tilting a dust-free disk away from face-on increases its apparent surface brightness)

(2) 2-D Velocity Fields :

a) For a flat disk, derive the coordinate transformations to go from: R, ϕ to *x'*, *y'* in the galaxy plane; and then to *x, y* in the sky plane after the galaxy is tipped by an angle *i* away from face on, such that the projected major axis is along the *x* axis.

b) What is the projected Doppler velocity for a point in the galaxy disk with circular velocity V_c and radial velocity V_r (+ve away from the nucleus), both in the plane of the galaxy?

c) On a simple 200 x 200 cartesian computational grid, use a contouring routine to generate "spider diagrams" (iso-projected velocity contours at intervals of 20 km/s) for the following velocity fields for a circular galaxy of radius 100 units, tipped through 60^o (label or color-code the contours, and indicate the zero velocity contour, which defines the kinematic minor axis) :

1. V_c = 150 (km/s) at all radii; V_r = 0 at all radii
2. V_c = solid body to R = 30, then flat at 150 km/s beyond; V_r = 0 at all radii
3. V_c = solid body to R = 30, then linear rise from 150 to 200 km/s; V_r = 0 at all radii
4. V_c = solid body to R = 30, then linear fall from 150 to 100 km/s; V_r = 0 at all radii
5. As for 4 but adding a uniform outflow of V_r = +50 km/s at all radii

d) For each of the velocity fields above, plot the integrated HI profile assuming that the galaxy fits within the HI telescope beam and has uniform surface HI density.

(3) Tully Fisher Relation :

1. Briefly outline a theoretical justification for expecting a relation between the luminosity of spiral galaxies and their rotation amplitudes of the form $L \propto V_{rot}^4$ (note: the same justification applies for the Faber-Jackson relation for spheroids: $L \propto \sigma^4$) . What additional constraints must hold for this relation to be valid?

2. What is the absolute magnitude of an exponential disk integrated out to a radius r if the disk has central surface brightness $\mu(0)$ mag/ss and scale length r_d pc. For a disk with $\mu_B(0)$ = 21.65 and r_d = 1.5 kpc, evaluate M_B :
   1. out to 1 r_d
   2. out to 2.2 r_d
   3. total (out to infinity)

3. Assume (without derivation) that the rotation curve for an exponential disk peaks at r_{max} = 2.2 r_d at a value $V_{max}^2 = G\ M(< r_{max}) / \beta\ r_{max}$ with

flattening factor β = 0.7 (see equation 5.6 in the notes). Assume further, that this maximum velocity is held constant to larger radii by the presence of a halo. Finally, assume that spirals obey the Tully Fisher relation: M_B = -7.41(log W - 2.5) - 20.04   (equation 5.5 in the notes).

Derive an expression for the M/L_B ratio for the disks of spirals out to radius r in terms of r_d and $\mu_B(0)$. What is the value at r = 2.2 r_d and 10 r_d for a disk with central surface brightness $\mu_B(0)$ = 21.65 and scale length r_d = 1.5 kpc. Comment on these two expressions and the difference between them.

(4) LOSVD's :

For an exactly edge-on pure stellar disk in which the stars all follow circular orbits at speed V_c, and in which the density of stars decreases with radius as $n(r) \propto r^{-2}$, show that the line-of-sight velocity dispersion (LOSVD) is given by : $F(V_{los} \propto (V_c^2 - V_{los}^2)^{-1/2}$ in the range $0 < |V_{los}| < V_c$ and zero otherwise. Calculate $< V_{los} >$; σ_{los}; ξ_3; and ξ_4 for such a disk.
