

# Whittle : EXTRAGALACTIC ASTRONOMY

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## 6. STELLAR DYNAMICS I : DISKS

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### (1) Introduction

#### (a) Scope of these Lectures

- We now begin a study of **Stellar Dynamics** : the theory of gravitational many-body systems. This subject is mature, extensive and can be highly sophisticated. Fortunately for us, we don't really need (and don't have the time for) a **detailed** treatment. Instead, my aim will be to :
  - present the major themes in broad outline
  - extract some important analytic results
  - develop our intuitive understanding of each topic
  - connect each topic to our observational knowledge of galaxies
- Since the subject is so weighty, it is sensible to spread it out a bit :
  - Topic 6 : considers disk dynamics and the formation of bars and spirals
  - Topic 8 : covers the meat of the subject : ways to understand the general 3-D system
  - Topic 12 : considers dynamical friction, encounters and mergers.
  - Topic 13 : discusses how black holes can affect galaxy structure

By the end we will have covered almost all of B&T while omitting much of the detail.

- So lets now begin with disk dynamics :  
The path for this topic takes us through a number of interesting themes :
  - epicycles** : perturbations of simple circular orbits
  - resonances** : when epicycles synchronize with the passage of disk patterns
  - density waves** : a self-consistent response to coherent epicyclic motion

**instabilities** : when disks begin to clump up under self-gravity  
**amplification** : when this clumping grows to form bars and spirals.



## (2) Epicycles

### (a) Overview

- Disk stars have approximately circular orbits with small deviations :  
 As with the Ptolemaic system : star orbits can be described by superposition of :
  - circular orbit along **guiding center** (=deferent), radius  $R_g$ , angular velocity  $\Omega_g$
  - smaller elliptical **epicycle**, angular velocity  $\kappa$ , **retrograde**
- consider gravity/centrifugal balance & conservation of AM :
  - consider star at guiding center (GC) and give it a kick radially **outwards**
  - conserving  $AM = mrv_\theta$ , since  $r$  increases,  $v_\theta$  decreases  
 → w.r.t. the guiding center, the star moves **backwards**
  - consider the new balance between gravity and centrifugal forces :  
 under AM conservation,  $F_{\text{centrifugal}} \propto v_\theta^2 / r \propto r^{-3}$  while  $F_{\text{grav}}$  falls more **slowly** than  $r^{-2}$   
 → at larger radii  $F_{\text{grav}} > F_{\text{centrifugal}}$  and the star gets pulled back **inwards**
  - as it falls in,  $r$  decreases so  $v$  increases and the star moves **forward** relative to the GC
  - but now  $F_{\text{centrifugal}} > F_{\text{grav}}$  and the star moves **outwards** again

→ the cycle repeats, and we have a small **retrograde epicycle** [ images ]  
 (An equivalent description considers the coriolis force in a rotating frame)
- In general  $\Omega_g$  and  $\kappa$  are different so orbits don't close
- However, when observed from frame rotating at  $\Omega_g - \frac{1}{2} \kappa$  :  
 → orbits are **closed ellipses**, centered on guiding center

At this point, lets briefly anticipate the relevance of epicycles for spiral arms :

- For epicycle phases which vary systematically with radius :  
 → nested elliptical orbits may crowd in a spiral pattern  
 this is called a **kinematic density wave**
- Orbits are in turn perturbed by the spiral density pattern,  
 this modifies the simple epicycles
- Self-consistent solution is a **density wave**  
 gas reponse causes shocks and star formation → visible spiral arms [ images ]

Now let's return to the epicycles, and derive their characteristics :

- Consider a smooth axisymmetric flattened mass distribution with potential  $\Phi(R,z)$   
 Since we have no azimuthal forces AM is conserved, and we have :

$$\ddot{\mathbf{r}} = -\nabla\Phi(R, z) ; \quad L_z = R^2\dot{\phi} = \text{const} \quad (6.1)$$

- Separating the Equations of motion into components in cylindrical coordinates  $(R, \Phi, z)$  :

$$\ddot{R} - R\dot{\phi}^2 = -\frac{\partial \Phi}{\partial R}; \quad \frac{d}{dt}(L_z) = 0; \quad \ddot{z} = -\frac{\partial \Phi}{\partial z} \quad (6.2)$$

## (b) Vertical z Motions

- Take first the z motion about the plane z=0  
since the disk is symmetric about z=0, then the z-force at z=0 is zero :

$$\left(\frac{\partial \Phi}{\partial z}\right)_{z=0} = 0 \quad (6.3)$$

consider small motion above and below the plane,  
we simply expand the z-force linearly for small z :

$$\ddot{z} = -\left(\frac{\partial \Phi}{\partial z}\right)_{z=0} - z\left(\frac{\partial^2 \Phi}{\partial z^2}\right)_{z=0} = -z\left(\frac{\partial^2 \Phi}{\partial z^2}\right)_{z=0} = -\nu^2 z \quad (6.4)$$

- This gives Simple Harmonic Motion (SHM), with frequency  $\nu$  where  $\nu^2 = (\partial^2 \Phi / \partial z^2)_{z=0}$  :

$$z(t) = Z \cos(\nu t + \psi_0) \quad (6.5)$$

- For Milky Way disk near the sun,  $\nu^2 = 4 \pi G \rho_0$ , which gives :  
→ vertical oscillation period  $2\pi / \nu \sim 6.5 \times 10^7 \text{ yr} \sim 1/3$  circular period,  $\Omega$ .

## (c) Radial Motions

- First consider the circular guiding orbit (R=const) :  
it has radius  $R_g$ , circular velocity  $V_c$  and angular velocity  $\Omega_g$  defined by

$$\left(\frac{\partial \Phi}{\partial R}\right)_{R_g} = \frac{V_c^2}{R_g} = R_g \Omega_g^2 \quad (6.6)$$

For non-circular orbits, the radial acceleration is given by (centrifugal - gravity) :

$$\ddot{R} = R\dot{\phi}^2 - \frac{\partial \Phi}{\partial R} \quad (6.7)$$

- However, notice that this can also be written as

$$\ddot{R} = -\frac{\partial \Phi_{\text{eff}}}{\partial R} \quad \text{where} \quad \Phi_{\text{eff}} = \Phi(R, z) + \frac{L_z^2}{2R^2} \quad (6.8)$$

Where the **effective potential**,  $\Phi_{\text{eff}}$ , allows us to describe the radial motion in 1-D form  
 Typically,  $\Phi_{\text{eff}}$  has a minimum, rising steeply at small R and slowly at large R [ images ]  
 This inner steep term imposes an **angular momentum (or centrifugal) barrier**  
 At the minimum in  $\Phi_{\text{eff}}$ , we recover the circular guiding orbit of radius  $R_g$

$$\left( \frac{\partial \Phi_{\text{eff}}}{\partial R} \right)_{R_g} = 0 = \left( \frac{\partial \Phi}{\partial R} \right)_{R_g} - R_g \dot{\phi}_g^2 = \left( \frac{\partial \Phi}{\partial R} \right)_{R_g} - \frac{V_c^2}{R_g} \quad (6.9)$$

- Other orbits will oscillate in radius, about  $R_g$ .  
 Consider the potential at  $R = R_g + x$

$$\ddot{R} = \ddot{x} = -\left( \frac{\partial \Phi_{\text{eff}}}{\partial R} \right)_{R_g} - x \left( \frac{\partial^2 \Phi_{\text{eff}}}{\partial R^2} \right)_{R_g} = -x \left( \frac{\partial^2 \Phi_{\text{eff}}}{\partial R^2} \right)_{R_g} = -\kappa^2 x \quad (6.10)$$

This gives SHM about the guiding radius

$$x(t) = X \cos(\kappa t + \phi_0) \quad (6.11)$$

with frequency  $\kappa$ , where

$$\kappa^2 = \left( \frac{\partial^2 \Phi_{\text{eff}}}{\partial R^2} \right)_{R_g} = \left( \frac{\partial}{\partial R} \left( \frac{\partial \Phi}{\partial R} \right) \right)_{R_g} + \frac{3L_z^2}{R_g^4} = \left( R \frac{d\Omega^2}{dR} + 4\Omega^2 \right)_{R_g} \quad (6.12)$$

#### (d) Azimuthal Motion

- Since  $L_z = R_g^2 \Omega_g = R^2 \Omega = \text{const}$ , changes in R yield changes in  $\Omega$  (recall  $\Omega = \dot{\phi}$ )

$$\dot{\phi} = \frac{L_z}{R^2} = \frac{L_z}{(R_g + x)^2} \simeq \frac{L_z}{R_g^2} \left( 1 - \frac{2x}{R_g} \right) = \Omega_g \left( 1 - \frac{2x}{R_g} \right) \quad (6.13)$$

Integration gives :

$$\phi(t) = \Omega_g t - \frac{2\Omega_g X}{\kappa R_g} \sin(\kappa t + \phi_0) \quad (6.14)$$

- Thus,  $\phi(t)$  follows the guiding center with small amplitude SHM superposed. Taking the y-axis in the forward direction with origin on the guiding center, we have

$$y(t) = -\frac{2\Omega_g}{\kappa} X \sin(\kappa t + \phi_0) \quad (6.15)$$

the oscillation of frequency  $\kappa$  is the same as in x, but out of phase by  $90^\circ$

Taken together, (and setting the initial phase  $\phi_0 = 0$ ), we have

- $x = X \cos(\kappa t)$
- $y = -(2\Omega/\kappa) X \sin(\kappa t)$
- **elliptical** epicycle with radial/azimuthal axis ratio =  $\kappa / 2\Omega$
- epicyclic motion is **retrograde** w.r.t. orbit (cf Ptolemy's were prograde)
- for stars of same  $R_g$ , velocity ellipsoid  $\sigma_R / \sigma_\theta = \kappa / 2\Omega$
- however, at given R, velocity ellipsoid  $\sigma_R / \sigma_\theta = 2\Omega / \kappa$
- for Keplerian :  $\Omega \propto R^{-3/2}$  we get  $\kappa = \Omega$   
closed ellipse, centered at the ellipse focus  
axis ratio 2:1 (cf Ptolemy's were 1:1 circles)
- For flat rotation curve  $\Omega \propto R^{-1}$  we get  $\kappa = \text{sqrt}(2) \Omega$
- for solid body rotation  $\Omega = \text{const}$  we get  $\kappa = 2\Omega$
- in general,  $\Omega < \kappa < 2\Omega$  so  
 $\kappa > \Omega$  and epicycle completed **before** rotation
- **Here** is a model of small amplitude epicyclic motion near the sun.

## (e) Values Near the Solar Neighborhood

It is useful to re-express  $\kappa$  and  $\Omega$  in terms of Oort's constants A and B :  
[a brief derivation and review of Oort's constants can be found [here](#)].

Oort's A expresses **local shear**

it is derived from radial velocities  $V_r / d = A \sin(2l)$  where  $l$ =longitude :

$$A = \frac{1}{2} \left( \frac{V_c}{R} - \frac{dV_c}{dR} \right)_{R_0} = -\frac{1}{2} R \left( \frac{d\Omega}{dR} \right)_{R_0} \quad (6.16a)$$

Current best (Hipparcos) estimates for Oort's A is  $14.8 \pm 0.8$  km/s/kpc (B&M p642)

Oort's B expresses **local vorticity**, ie local rotation,  $\nabla \times \mathbf{V}$   
it is derived from proper motions  $V_t / d = A \cos(2l) + B$

$$B = -\frac{1}{2} \left( \frac{V_c}{R} + \frac{dV_c}{dR} \right)_{R_0} = -\frac{1}{2} R \left( \frac{d\Omega}{dR} + \frac{2\Omega}{R} \right)_{R_0} \quad (6.16b)$$

Current best estimate for Oort's B is  $-12.4 \pm 0.6$  km/s/kpc

Together these give :

$$\Omega_0 = A - B = 27 \text{ km/s/kpc} \quad (6.17a)$$

$$\kappa_0^2 = -4B(A - B) = -4B\Omega_0 ; \kappa_0 = 37 \text{ km/s/kpc} \quad (6.17b)$$

$$\frac{\kappa_0}{\Omega_0} = 2 \sqrt{\frac{B}{A - B}} = 1.3 \quad (6.17c)$$

- Stars in the solar neighborhood make 1.3 epicyclic excursions per orbit.
- From disk mass models, we have a z-force law shown [here](#)
- for peculiar velocity  $\sigma_R \sim \sigma_z \sim 30$  km/s we have :
  - epicycle sizes  $\sim \sigma / \kappa \sim 1$  kpc
  - vertical excursions  $\sim \sigma / \nu \sim 500$  pc
  - (SHM might not be quite valid for vertical motion)
- for the Sun,  $W = 7$  km/s (z direction), giving the excursions shown [here](#)  
 Note the sun passes through the disk plane every  $\sim 42$  Myr  
 there is evidence that species extinction and cratering has a similar period ( $\sim 36$  Myr)  
 tidal perturbations to the Oort cloud may be responsible : ([simulation](#))

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### (3) Resonances

#### (a) Rotating Patterns

- As we shall see (§ 4 below) there can be regions of **enhanced stellar density** in the disk these can have the shape of spiral and bar **patterns**  
 these patterns are neither stationary nor move with the disk stars,  
 instead they move at some intermediate angular velocity,  $\Omega_p$ , called the **pattern speed**  
 the interaction of these patterns with the epicyclic motion can lead to **resonances** :

#### (b) Corotation Resonance (CR)

- First consider a star which orbits **at the pattern speed**  
 we have  $\Omega_* = \Omega_p$  or  $\Omega_p - \Omega_* = 0$

- such stars experience a **persistant** non-axisymmetric perturbation  
their response can build up (epicycle phase rotates)  
(technically, this is a resonance to azimuthal epicycle perturbation, with natural frequency 0)

## (c) Lindblad Resonances (ILR, OLR)

- Consider stars which complete exactly 1 epicycle between the passage of each arm
- their interaction with the spiral arm is **resonant**  
epicyclic amplitude is amplified and wave propagation is strongly modified
- where do such resonances occur in the galaxy ?  
the angular frequency of the star w.r.t. the pattern is  $\Omega_p - \Omega_*$   
there are two cases :  
-ve if  $\Omega_* > \Omega_p$  ; star moves past arms  
+ve if  $\Omega_p > \Omega_*$  ; arms move past stars  
angular frequency of encountering **each** arm in an m armed spiral is  $m(\Omega_p - \Omega_*)$   
so condition for resonance is :  
 $m(\Omega_p - \Omega_*) = \pm \kappa$  or  $\Omega_p - \Omega_* = \pm \kappa / m$   
(notice that  $\Omega_p - \Omega_* = -\frac{1}{2} \kappa$  is a special case of a two armed spiral with  $\Omega_p < \Omega_*$ )
- there are two classes of resonance :  
 $\Omega_p - \Omega_* = -\kappa / m$  : **Inner Lindblad Resonance**  
 $\Omega_p - \Omega_* = +\kappa / m$  : **Outer Lindblad Resonance**
- to establish the radii of these resonances, one needs to know :  
the pattern speed :  $\Omega_p$   
the rotation curve,  $V_c(R)$ , which gives  $\Omega(R)$  and  $\kappa(R)$   
depending on  $V_c(R)$  and  $\Omega_p$  there may be 0,1,2,... resonances [ images ]  
(note : there can be 0,1,2 **inner** Lindblad resonances)

## (d) Importance of Resonances

Resonances are important for several reasons :

- density waves can only survive inbetween the ILR and OLR  
the forcing function must be **slower** than  $\kappa$  to sustain the wave
- density waves cannot pass an ILR  
they are absorbed, like waves on a beach  
important in allowing/preventing propagation across the disk through the center
- orbit **shapes** change across the resonances :  
forcing below/above natural frequency yields in/out phase response :  
outside ILR,  $x_1$  orbits parallel to bar  
inside ILR,  $x_2$  orbits perpendicular to bar  
outside CR orbits again perpendicular  
→ cf near nuclear orthogonal bars are common  
→ cf bars don't extend beyond CR, stop close to it  
→ bars probably rotate with pattern speed  $\Omega_p \sim \Omega(R=CR)$
- expect stellar **rings** to form at CR and OLR  
→ inner and outer rings do seem to correspond to expected CR/OLR locations
- gas driven **inwards** to ILR and **outwards** to OLR  
expect (and find) gas rings/disks/starformation near ILR
- gas only runs **into** arm at  $R < CR$   
expect dust lanes and star formation ridges to fade out close to CR
- note that for the Milky Way, estimates are (for  $m=2$ ) :

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## (4) Density Waves

### (a) Kinematic Density Waves

- In general, orbits are **not closed** (~1-2 epicycles per orbit)
- However, consider a frame which rotates at  $\Omega_g - \frac{1}{2} \kappa$ 
  - at  $\Omega_g$  we move **with** the guide center  
we witness just the epicycle with no orbital motion
  - now go slower by  $\frac{1}{2} \kappa$  ;  
after 1 epicycle, star moved through half an orbit  
after second epicycle back to start
  - orbit is **closed** in this frame
  - orbit is ellipse with GC at **center**
  - in general,  $\Omega - n\kappa/m$  will close after  $m$  radial oscillations [ images ]
- Now consider set of orbits whose epicyclic phases vary **monotonically** with radius (ie PA of ellipses rotates with increasing R)
- simple **orbit crowding** will generate a two arm spiral pattern (viewgraph)
- roughly, if  $\Omega - \frac{1}{2} \kappa \sim \text{const}$  for all radii, then pattern  $\sim$  **fixed** (in rotating frame)  
in non-rotating frame, pattern rotates at  $\Omega_p = \Omega_g - \frac{1}{2} \kappa =$  **pattern speed**  
for observed  $V(R)$ ,  $\Omega_g - \frac{1}{2} \kappa$  is fairly constant, so pattern is quite long lived
- for ~flat rotation curve, however, pattern speed does vary slowly with R, so spiral **winds up**  
but slower than "material" windup by factor  $\sim \Omega_p / \Omega \sim 0.3$   
some improvement, but still not adequate
- However, including **self-gravity** can yield a **different**  $\Omega_p$  which is almost **independent of radius**  
this is a result from "density wave theory", to which we now turn :

### (b) Lin-Shu (QSSS) Density Wave Theory

#### (i) Motivation and Sketch of Approach

The generation of **kinematic** spirals assumes **axisymmetric** potential  
However, orbit crowding yields a **non-axisymmetric** spiral perturbation  
Star and gas orbits **modified** by the spiral perturbation  
Their new orbits define a **new** surface density and associated potential  
We need to look for a **self consistent** solution : response to input potential gives **same** potential  
The analysis is difficult (Lin & Shu 1964, 1966 and much subsequent work)

Look for QSSS (Quasi Stationary Sprial Structure) -- ie  $\Omega_p(R) \sim \text{const}$

Study response of epicycles to periodic forcing function (as they pass the spiral arms)

This is re-cast as **waves** propagating in a differentially rotating disk

Analytic methods only work for tightly wound spirals (WKB methods)

Derive a **dispersion relation** :  $w = f(k)$  giving

phase velocity =  $w / k$  and group velocity  $dw / dk$   
( $w$ =frequency,  $k$ =wavenumber)

difficulties are encountered near the ILR, OLR, CR, but they are tractable

#### (ii) Results

- QSSS solutions are found with  $\Omega_p = \Omega - (dw/dk) \times \frac{1}{2}\kappa \sim$  independent of R
- pitch angles  $\psi(R) \sim \text{const}$ , yielding logarithmic spirals
- waves sustained if forcing frequency  $m|\Omega_p - \Omega(R)|$  is slower than  $\kappa(R)$   
this occurs between ILR and OLR  
note : this region is larger for  $m=2$  than  $m=4$ , so **two** spirals most common
- waves are **absorbed** at ILR
- wave response is reduced for disks with higher velocity dispersion  
probably need a "cold" component to be continually replenished
- response of **gas** is non-linear (collisions/shocks) above threshold [ images ] :  
 $k^2 F / |\kappa^2 - w^2| > 1$   
where spiral potential is  $\Phi(R, \phi) = F \cos(\kappa R + m\phi)$  with  $w =$  forcing frequency =  $m(\Omega - \Omega_p)$
- gas runs into itself (cf traffic jams) :  
expect narrow **gas** features --- as observed
- predict **velocity streaming** in vicinity of arms :  
calculations (when tweaked) seem to fit well, cf HI study of M81 : [ images ]
- explains the fundamental correlation between Bulge/Disk ratio and Pitch angle in Spirals :  
Roberts, Roberts & Shu (1975) :  
show geometry of density wave and strength of shock depend on :
  - (a) galaxy mass/size ( $\sim V_{\text{rot}}$ ), and
  - (b) central concentration.
 This explains :
  - (a) correlation of pitch angle with B/D ratio
  - (b) at given stage, pitch angle increases with lower luminosity gals
  - (c) absence of dwarf spirals since in low mass galaxies the spiral shock is weaker/absent, leading to dwarf irregulars.

### (iii) Remaining Problems with QSSS

despite much theoretical effort, still unclear if Lin-Shu QSSS important for real galaxies

- astrophysical complexities :  
cold gas only 2-4% of volume (hot gas won't respond)  
... something else here ?
- unclear whether **strength** of many observed arms can be achieved
- passage through arms increases dispersion (and therefore Q)  
this will quench the arms and halt the process  
unclear whether creation of new cold component is sufficient to maintain process
- Careful study of  $\sim 50$  galaxies (Kormendy & Norman) reveal **few** which may need QSSS :
  - Many galaxies have **other** causes of density waves (eg tides, bars, ovals -- see below)
  - Many (most) galaxies are **flocculent** with no obvious **global** spiral pattern
  - Yet other galaxies (eg late Sc) have large solid body rotation region  
coherent **material** arms can exist here --- no need for density wave.

## (c) Alternative Sources of Global Density Waves

There are two other obvious sources of density waves (both  $m=2$ )

### (i) Tides

- tidal field of passing neighbor creates a strong  $m=2$  perturbation
- early simulations (eg Toomre & Toomre 1972) nicely show strong outer arms  
arms did not, however, go to center (viewgraph)
- later simulations (Toomre 1981) included self-gravity and did better [ images ]
- nowadays, many simulations reveal a strong deep  $m=2$  spiral
- note, however, these spirals are **transient**

## (ii) Bars and Oval Distortions

- bars are another source of  $m=2$  perturbations [ images ]
- Sanders & Huntley (1976) found a weak bar generates strong spiral arms
- However, needs **viscosity** (ie dissipation)  
→ removing the viscosity stops the arm formation
- conclusion : spiral arms only need a bar and dissipation to form

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## (5) Disk Instabilities and Their Amplification

### (a) Local Disk Stability : Toomre's Q Parameter

When are self-gravitating disks vulnerable to **local** gravitational instabilities ?  
Such instabilities, together with differential rotation, may generate spiral structure

- Instabilities can arise from a competition between :
  - gravity causing overdense regions to collapse
  - stellar dispersion which inhibits the collapse
  - angular momentum which inhibits the collapse
- Toomre (1964) found the conditions for instabilities :  $Q < 1$  where  
for gas disk :  $Q = \kappa V_s / (\pi G \Sigma)$   
for stellar disk :  $Q = \kappa \sigma_R / (3.36 G \Sigma)$   
where  $V_s$  is the sound speed and  $\Sigma$  is the local surface density

Lets look briefly at the derivation :

### (i) Modified Jeans Analysis

Consider overdense region radius  $R$  in a **non-rotating** disk

- the collapse time is  $t_{\text{coll}} \sim R / V$  where  $V \sim$  gravitational velocity  $\sim (G M / R)^{1/2}$   
so  $t_{\text{coll}} \sim R / (GM / R)^{1/2} \sim (R^3 / GM)^{1/2} \sim (R / G \Sigma)^{1/2}$  ( $\Sigma$  is surface density)  
the time for stars to escape the region is :  $t_{\text{esc}} \sim R / \sigma$  ( $\sigma$  is dispersion)  
so collapse occurs if  $t_{\text{coll}} < t_{\text{esc}}$  ie  $(R / G \Sigma)^{1/2} < R / \sigma$

→ The critical size **for stability** due to dispersion is therefore :  $R_J < \sigma^2 / G \Sigma$

- Now consider a **rotating** disk  
the local angular velocity is Oort's constant  $B$   
the region is **stable** if  $F_{\text{centrifugal}} > F_{\text{gravity}}$   
in this case  $R B^2 > GM / R^2 = G \Sigma$

→ The critical size **for stability** due to rotation is therefore :  $R_{\text{rot}} > G \Sigma / B^2$

- Combining these : the disk is **unstable** in the range  $R_J < R < R_{\text{rot}}$   
and therefore the disk is locally **stable** if  $R_J > R_{\text{rot}}$   
ie  $\sigma^2 / G \Sigma > G \Sigma / B^2$  or  $\sigma B / G \Sigma > 1$

recall that  $B = \kappa^2 / 4 \Omega$  and  $\kappa \sim 1-2 \Omega$  so  $B \sim \kappa / 3$

The final condition for disk stability is therefore :  $Q \equiv \sigma \kappa / 3 G \Sigma > 1$

## (ii) Q from Disk Dispersion Relations

consider the time development of wave perturbations in a disk :  $\delta\rho \propto \exp i(\mathbf{k} \cdot \mathbf{r} - \omega t)$   
Solve the Collisionless Boltzmann Equation and extract the dispersion relation :

$$\omega^2 = \kappa^2 - 2\pi G \Sigma |k| + k^2 V_s^2 \quad (\text{Gas}) \quad (6.18a)$$

$$\omega^2 = \kappa^2 - 2\pi G \Sigma |k| \cdot F(\Omega, k, \sigma_R) \quad (\text{Stars}) \quad (6.18b)$$

where  $V_s$  = sound speed, and F is a "reduction factor" (see B&T § 6.2d)

the disk is **unstable** if  $\omega^2 < 0$  for all k, since  $\exp(-i\omega t) = \exp(\gamma t)$  with  $\gamma$  real

This yields for stability :

$$Q_{\text{gas}} \equiv \kappa V_s / \pi G \Sigma > 1$$

$$Q_{\text{stars}} \equiv \kappa \sigma_R / 3.36 G \Sigma > 1$$

- Factors which promote gravitational instability are :  
low stellar velocity dispersion  $\sigma_R$   
high surface mass density  $\Sigma$   
low epicyclic frequencies
- Note : even for  $Q > 1$ , disks may still be vulnerable to **global** instabilities (modes)
- Solar neighborhood :  $\sigma_R \sim 30$  km/s ;  $\Sigma \sim 50 M_{\odot} \text{pc}^{-2}$  ;  $\kappa \sim 36$  km/s/kpc  
these give  $Q \sim 1.4$  and so the MW disk is locally stable near the sun

## (b) Swing Amplification

Some circumstances allow a powerful amplification of spiral patterns (eg Toomre 1981)

### (i) The Swing Amplifier

If we have a **leading** spiral density wave, then

- differential rotation will gradually rotate it into a **trailing** spiral wave [ images ]
- the rotation of the pattern is **retrograde**
- the timescale for rotation is  $\sim \kappa$   
→ epicyclic motion approximately **follows** the arm  
→ **long** perturbation duration so epicycle amplified  
→ the emerging trailing pattern is strongly **amplified**
- amplification **gain** depends on Q and  $\lambda$  (radial wavelength of pattern) [ images ]  
maximum  $\sim 10^2$  when  $\lambda \sim 1.5 \lambda_{\text{crit}}$ , where  
 $\lambda_{\text{crit}} = 4 \pi G \Sigma / \kappa^2$  = shortest  $\lambda$  normally stabilized by rotation  
for  $\lambda > 2 \lambda_{\text{crit}}$  less effective; stabilized by rotation
- just disk noise has all  $\lambda$  present  
→  $\lambda \sim 1.5 \lambda_{\text{crit}}$  amplified [ images ]
- as usual, subsequent disk heating raises Q and suppresses the amplification mechanism

This **swing amplification** is thought to be very important

## (ii) Feedback for the Amplifier

For this to work, we need a source of **leading** spiral waves however, these are not normally generated in a rotating disk  
Instead, look for **feedback** : trailing waves converted into leading waves.

- waves **reflected** from outer edge experience  $180^\circ$  phase shift (trailing  $\rightarrow$  leading)  
unlikely to operate in real galaxies, edges too soft
- trailing waves passing through the center emerge as leading waves [ images ]  
this can only occur when we have no ILR (which blocks wave passage)

**Swing amplification with feedback is probably very important in maintaining strong spiral structure.**

## (c) Bar Instability and its Suppression

- N-Body simulations of disks seem to form bars **remarkably easily**  
Indeed, it is difficult to devise stable disk models **even with  $Q > 1$**   
Reality of this **bar instability** has been verified using analytic methods  
Here is an N-Body simulation showing bar formation : [image]  
multi-arm spirals form first, then two arms, then a bar  
the bar lasts until the end of the simulation  
the instability can be suppressed with **high dispersion** (as for spiral density waves)
- Swing Amplification helps explain the instability :  
recall : leading waves are strongly amplified into trailing ones  
nothing happens unless there is a source of leading waves  
trailing waves pass through center and emerge as leading  
hence **feedback** keeps the amplifier going  
 $\rightarrow$  bar grows quickly.
- recall, waves cannot pass an ILR where  $\Omega - \Omega_b = -\kappa/2$   
so for feedback to work : need  $\Omega_b > \Omega - \kappa/2$  everywhere  
simulations verify this and "confirm" central tunneling as a source of feedback
- Equivalent description involves epicyclic response to forcing function  
if  $\Omega - \Omega_b < -\kappa/2$  then the forcing frequency is **greater** than the epicyclic frequency  
 $\rightarrow$  response is out of phase by  $\pi$   
 $\rightarrow$  feeds  $x_1$  orbits which extend **along** the bar  
 $\rightarrow$  bar can grow by accululating stars in  $x_1$  orbits  
  
However, if ILR exists,  $\Omega - \Omega_b < \kappa/2$   
 $\rightarrow$  response is **in phase**  
 $\rightarrow$  favours  $x_2$  orbits which are **perpendicular** to the bar  
 $\rightarrow$  weakens the bar  
Likewise, outside co-rotation,  $\Omega - \Omega_b < \kappa/2$   
 $\rightarrow$   $x_1$  orbits again suppressed while  $x_2$  orbits reinforced  
 $\rightarrow$  explains why bars rarely extend beyond CR
- Early work (Hohl 1971, Ostriker & Peebles 1973) noted the severity of the bar instability  
as usual, increasing stellar **dispersion** can calm the instability  
they found disks were stabilized against bar formation for  $KE(\sigma) / KE(\text{rot}) > 5$

Note that for the MW disk near the sun,  $KE(\sigma)/KE(\text{rot}) \sim 0.15$   
→ so our disk should be massively unstable to bar formation !

- What might suppress the bar instability in real galaxies ?  
there are several likely mechanisms :
  - put mass in a dark halo : this acts like a high dispersion component  
probably more of historical interest :  
back in 1973, any evidence for a dark halo was promoted  
however : dynamics of the inner regions are **not** influenced much by the halo  
the halo may nevertheless help stabilize disks at larger radii
  - achieve the same by having a high dispersion bulge or inner disk  
ingoing waves are damped before they pass through the center : cuts feedback
  - an ILR will shield the center and cut the feedback :  
a large central bulge mass will yield an  $\Omega(R)$  with an ILR

## (d) Flocculant Spirals

Recall that many spirals do **not** have strong density waves [image]  
these may have a different cause of spiral arms :  
→ patchy star formation smeared by differential rotation  
→ dynamic process : arms come and go

Two processes are discussed :

### (i) Local Disk Instability

for disks with  $Q_{\text{gas}} \lesssim 1$ , gravity clumping will occur

for  $Q_{\text{gas}} \sim 1$  the most unstable wavelength is  $\sim 2 \pi G \Sigma_{\text{gas}} / \kappa^2 \sim 200$  pc

expected feature scale size  $\sim 0.2$  kpc, as observed

this may be **self-regulating** : SN 'heats' gas; increases  $\sigma_{\text{gas}}$  and  $Q_{\text{gas}}$  → stability

### (ii) Self-Propagating Star Formation

Idea introduced by Mueller and Arnett (1976) :

SN & winds from star formation trigger further adjacent star formation  
region is stretched by differential rotation → spiral like patterns

for coherent **spiral detonation wave**, can yield **fixed pattern**

Problems : models need fine ( $\sim 10\%$ ) tuning to get good arms

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