Circular Galactic Rotation.
Derivation of Oort's constants: A & B

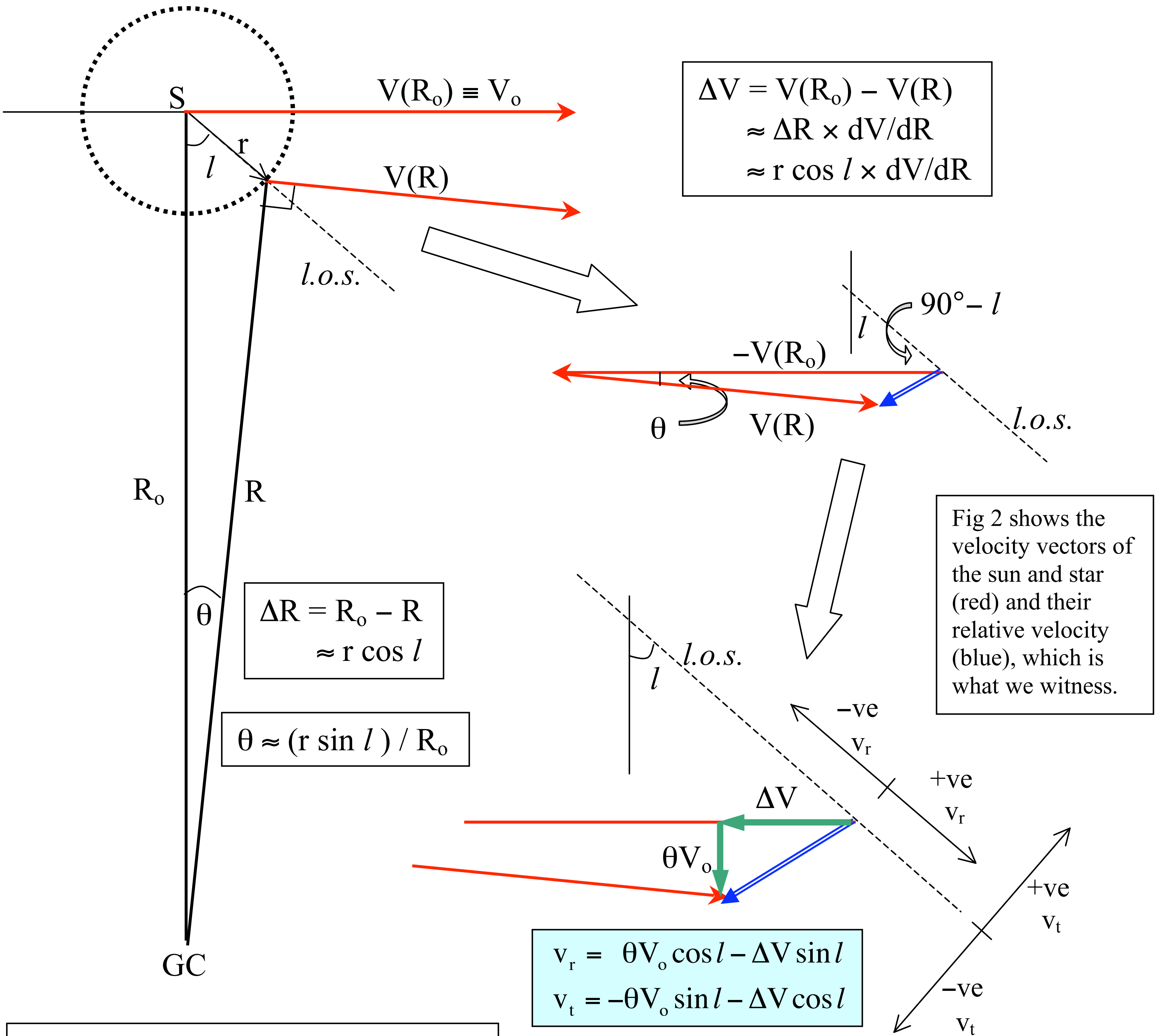


Fig 2 shows the velocity vectors of the sun and star (red) and their relative velocity (blue), which is what we witness.

Fig 1 shows the global situation: a circular rotation velocity field of amplitude V(R) about the GC with the sun at S. Consider a star at galactic longitude l, distance r from the sun: it subtends $\theta \approx (r \sin l) / R_o$ at the GC, and its velocity also subtends θ w.r.t. the solar velocity. It is closer to the GC by $\Delta R \approx r \cos l$ and differs in V by $\Delta R \times dV/dR$.

Fig 3 zooms into the difference velocity (blue). We want to find its projection parallel (v_r: doppler) and transverse (v_t : proper motion) to the line of sight (*l.o.s.*). To do this, make use of its (green) components parallel and perpendicular to the solar velocity (amplitudes ΔV and θV_o).

Radial velocity components projected onto the line of sight:

$$\boxed{v_r} = \theta V_o \cos l \;-\; \Delta V \sin l \;=\; \frac{V_o}{R_o}\, r \sin l \cos l \;-\; \frac{dV}{dR}\, r \sin l \cos l$$

$$= r \sin l \cos l \left(\frac{V_o}{R_o} - \frac{dV}{dR} \right) \;=\; \boxed{A\, r \sin 2l}$$

$$\text{where } A = \tfrac{1}{2}\left(\frac{V_o}{R_o} - \frac{dV}{dR} \right)_{R_o} \quad \text{is Oort's first (sheer) constant.}$$

Transverse velocity components projected onto the sky (proper motion):

$$\boxed{v_t} = -\theta V_o \sin l \;-\; \Delta V \cos l \;=\; -\frac{V_o}{R_o} r \sin^2 l \;-\; \frac{dV}{dR} r \cos^2 l$$

$$= -\tfrac{1}{2}\left(\frac{dV}{dR} + \frac{V_o}{R_o} \right) r\,(\cos^2 l + \sin^2 l) \;-\; \tfrac{1}{2}\left(\frac{dV}{dR} - \frac{V_o}{R_o} \right) r\,(\cos^2 l - \sin^2 l)$$

$$= -\tfrac{1}{2}\left(\frac{dV}{dR} + \frac{V_o}{R_o} \right) r \;+\; \tfrac{1}{2}\left(\frac{V_o}{R_o} - \frac{dV}{dR} \right) r \cos 2l \;=\; \boxed{B\, r \;+\; A\, r \cos 2l}$$

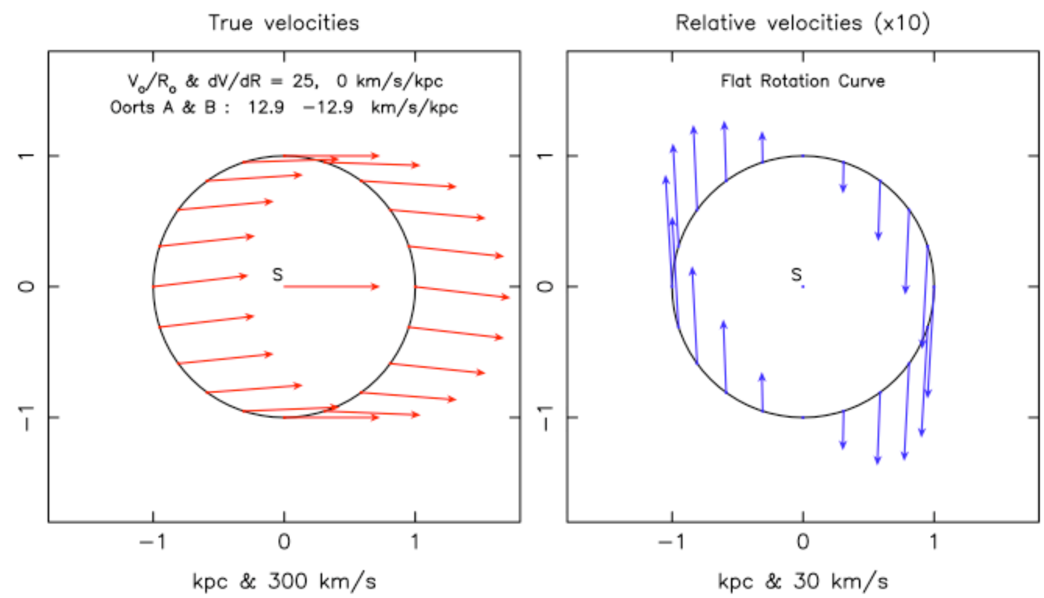
$$\text{where } B = -\tfrac{1}{2}\left(\frac{V_o}{R_o} + \frac{dV}{dR} \right)_{R_o} \quad \text{is Oort's second (rotation) constant.}$$

A measures the local sheer: the degree to which stars slide past each other. E.g. solid body has no sheer, since $dV/dR = V/R$, so $A = 0$. Note a flat rotation curve ($dV/dR = 0$) does have sheer, with $A = \frac{1}{2} V/R$.

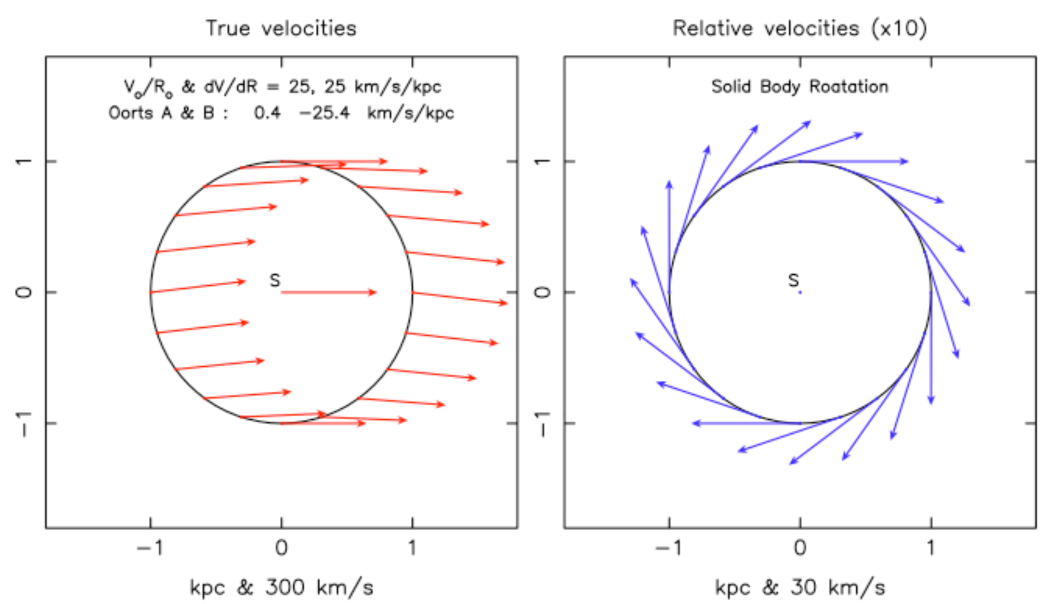
B measures local rotation, or vorticity. It comes from the curl of the velocity field: $B = \frac{1}{2} \nabla\times \mathbf{V}$. E.g. solid body is pure rotation, at the angular velocity of the disk: $B = -V/R$.

Here are some examples of circular disk rotation, with full velocities shown on the left (red vectors), and the differential velocity on the right (blue vectors). Each pair is for a particular rotation curve gradient near the sun (dV/dR). The galactic center is at y = –8.5 kpc, and V_{sun} = 220 km/s. The circle is 1 kpc in radius.

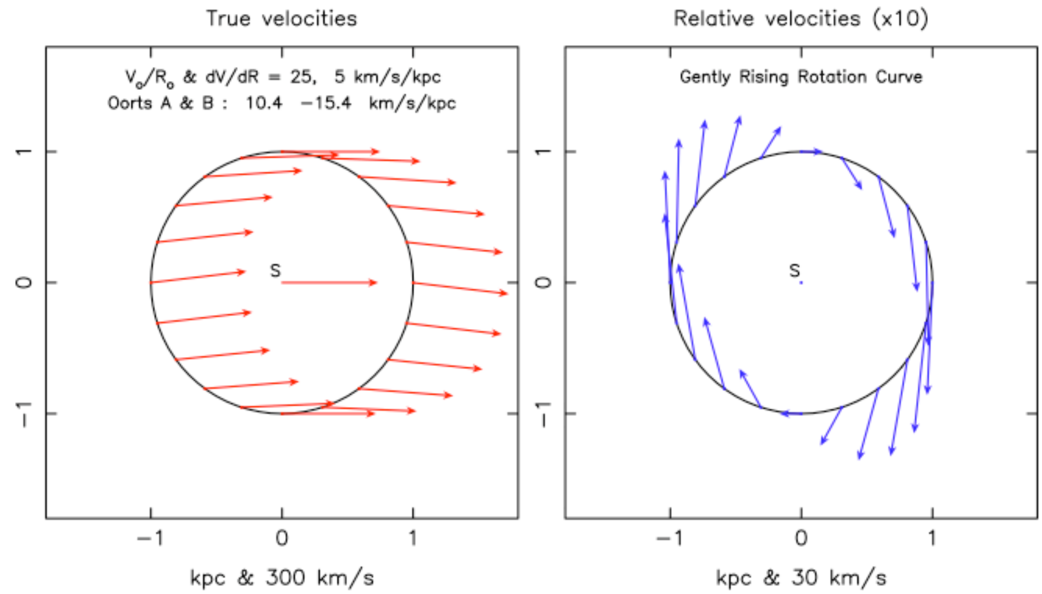
Flat Rotation Curve



Solid Body Rotation Curve



Gently Rising Rotation Curve (+5 km/s/kpc)



Gently Falling Rotation Curve (−5 km/s/kpc)