Selection of Homework Questions

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Topic 6: Theory I : Disks

(1) Epicyclic Motion : Theory

a. Derive the radial oscillation frequency, κ, for a star perturbed from a circular orbit in an arbitrary axisymmetric potential $\Phi(R)$. Express your result first in terms of the angular velocity, $\Omega(R)$, and then in terms of the rotation curve, $V(R)$.
b. Show that a disk in which the angular momentum (per unit mass) decreases outwards cannot support stable circular rotation. [Hint: find the condition that perturbations to circular motion **cannot** yield small epicyclic oscillations.] (B&T-2 Q 3.8)
c. Starting with Poisson's equation in cylindrical coordinates: $\nabla^2 \Phi = 4 \pi G \rho$ (see B&T-2 Eq B.52 p 777), show that an axisymmetric galaxy has epicycle, vertical and orbital frequencies which obey: $\kappa^2 + \nu^2 - 2 \Omega^2 = 4 \pi G \rho$.
d. Use solar neighborhood values for κ, ν, and Ω, to estimate the local density in the MW disk. (Adapted from B&T-2 Q 3.15).

(2) Solar Epicyclic Motion :

For the sun, assume a current galactocentric distance $R_\odot$ = 8.5 kpc; Oort's constants A = 15 km/s/kpc and B = -12 km/s/kpc; and a current solar motion relative to the local circular velocity of V_r = -10 km/s (ie towards the galactic center) and V_ϕ = +5.2 km/s (ie faster than circular).

a. Using the epicycle approximation, what are the Sun's minimum and maximum distances from the Galactic center ?
b. Assuming the Sun currently resides in the plane and has V_z = 7 km/s, what is the maximum excursion above and below the plane (assume a local mass density of 0.2 $M_\odot\ pc^{-3}$, which extends well above the excursion height).

(3) Disk Resonances :

a. Use psm units (Topic 1.3e) to quickly show that a velocity gradient of Ω km/s/kpc has associated angular velocity Ω radians/Gyr, frequency $\Omega/2\pi$ Gyr^{-1}, and period $P = 2\pi / \Omega$ Gyr.
b. Consider circular orbital motion of angular velocity Ω viewed in a frame rotating with angular velocity F (same, CCW, direction). What is the **apparent** angular velocity and period of the star? Now add retrograde epicyclic motion of angular velocity κ. For what values of F does the new orbit appear closed? Sketch the shape of the orbit as seen from the rotating frame when F is:
   1. $\Omega - \kappa$
   2. $\Omega - ½ \kappa$
   3. $\Omega - {}^{1}/_{3} \kappa$
   4. $\Omega + ½ \kappa$
   5. $\Omega - 0.49 \kappa$

   Consider a three armed spiral with pattern angular velocity $\Omega_p = \Omega - {}^{1}/_{3} \kappa$. How does the star's epicyclic motion interact with the pattern?
c. A galaxy has a rotation curve with a solid body inner part, and slowly falling outer part:
   $V_c = 100$ R km/s, $0 < R < 3$ kpc
   $V_c = 330 - 10$ R km/s, $R > 3$ kpc.
   The galaxy has a bar and spiral pattern which have constant slow angular velocity of 20 km/s/kpc.

   On a single plot, show and label clearly the following functions of R: Ω; $\Omega - ½\kappa$; $\Omega + ½\kappa$; Ω_p. On a plot with the same x-axis (but with different y-axis), show the rotation curve, V(R). [Hint: it is easiest to evaluate κ(R) numerically rather than algebraically].

d. Identify, if present, the locations of the ILR, CR and OLR resonances.

(4) Estimating Pattern Speeds : Express all frequencies in km/s/kpc, and in Myr^{-1}

a. For a galaxy with a flat rotation curve at 250 km/s, what's the epicyclic frequency at R = 7 kpc?
b. If corotation is at R = 6 kpc, what's this galaxy's pattern speed ?

c. For a two-armed spiral, is R = 7 kpc a resonance radius ?
d. Assume the outer Lindblad resonance is at R = 20 kpc. What's the galaxy's pattern speed now (assume the pattern has m = 2) ?

(5) Disk Stability :

a. Derive an approximate expression for local disk instability to gravitational clumping, the so-called Toomre Q parameter (for stars).
b. A galaxy has solid body rotation ($V = 100 \times R_{kpc}$) out to 3 kpc, and falls slowly ($V = 330 - 10\ R_{kpc}$) beyond. The disk itself has an exponential scale length of 3 kpc, and surface mass density of 100 $M_\odot\ pc^{-2}$ at 6 kpc. Assume the disk has uniform velocity dispersion σ = 20 km/s and uniform M/L ratio (i.e. the surface density is also exponential).

   Use Toomre's criterion to find which parts of the disk are locally unstable (hint: it is probably easiest to evaluate Q numerically).

c. If the disk is "heated" by the passage of orbiting satellites, what is the minimum value of σ that will supress local instabilities (and associated star formation) throughout the disk?
