

Entropy Evolution of the Gas in Cooling Flow Clusters

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We emphasize the importance of the gas entropy in studying the evolution of cluster gas evolving under the influence of radiative cooling. On this basis, we develop an analytical model for this evolution. We then show that the assumptions needed for such a model are consistent with a numerical solution of the same equations. We postulate that the passive cooling phase ends when the central gas temperature falls to very low values. It follows a phase during which an unspecified mechanism heats the cluster gas. We show that in such a scenario the small number of clusters containing gas with temperatures below about 1 keV is simply a consequence of the radiative cooling.

1. Introduction

Various speakers at this meeting have shown overwhelming evidence that to explain our observations of the hot gas in galaxy clusters we need to invoke some form of non-gravitational heating of this gas. Various mechanisms for heating the gas in galaxy clusters have been proposed. All of these can be divided into two categories: Steady-state or episodic heating scenarios. In the steady-state picture the heating mechanism ‘knows’ locally about the radiative cooling. Every bit of energy radiated away by the gas at a given place inside the cluster is replaced by an equal bit of energy at the same place by the heating process. The cluster as a whole never changes. For episodic heating this tight spatial and temporal connection between cooling and heating is broken. The cluster gas spends some time passively cooling and the lost energy is then replenished during a heating period. The energy input does not have to be distributed in this case. For example, the heating of the cluster gas by an AGN outflow may initially only affect the very center of the cluster. However, such a localized release of energy adds the further complication of having to subsequently distribute the energy throughout at least the inner volume of the cluster. This distribution of energy cannot be achieved by a fully convective flow as this would imply a negative entropy gradient in the cluster which has never been observed.

In this contribution we are not investigating any heating mechanisms. We simply assume that the cluster gas is heated in an episodic fashion, possibly by AGN outflows. In between heating phases the cluster gas cools passively until the temperature of the gas at the cluster center vanishes. At this point the heating mechanism is triggered once more. In the following we solve the hydrodynamical equations governing the evolution of the cluster gas during the passive cooling periods. We will show that the properties of the cluster gas predicted by our model are consistent with observations. In particular we demonstrate that our model can explain why we find very few clusters containing gas at temperatures below

about 1 keV.

In section 2 we motivate our approach of concentrating on the entropy of the cluster gas rather than its internal energy. Section 3 outlines the development of a simple analytical model describing the evolution of the cluster gas. The assumptions for the analytical model are tested against a numerical solution of the same equations in section 4. Finally, in section 5 we use the numerical approach to demonstrate that passive radiative cooling alone can explain the apparent paucity of detections of clusters containing very cold gas.

2. Why Is Entropy Important?

In the following we give two reasons why the entropy of a gas is an important quantity to study in galaxy clusters. The first is a significant simplification of the hydrodynamic equations describing the cooling flow gas. Secondly, observations of cluster gas reveal a simple power-law relation between the gas’ entropy and its mass.

2.1. Comparison With Energy

The properties of the hot gas in galaxy clusters evolving under the influence of radiative cooling are traditionally described by a set of three hydrodynamical equations: Conservation of mass, momentum and energy (e.g. Sarazin 1988). Usually these equations are then solved numerically and the resulting inflow of gas towards the cluster center due to radiative cooling is referred to as a cooling flow. It is intuitive to use the conservation of energy equation as the rate of change of the internal energy of the gas due to radiative cooling is very simple, i.e.

$$\frac{\partial E_{\text{rad}}}{\partial t} \propto \rho^2 \Lambda(T), \quad (1)$$

where ρ is the mass density and $\Lambda(T)$ is the temperature-dependent cooling function. However, the total rate of change of the internal energy is much more complicated. As the gas cools, it is compressed by the somewhat hotter gas further out in the cluster. This adiabatic work increases the internal energy. Radiative cooling is a strong

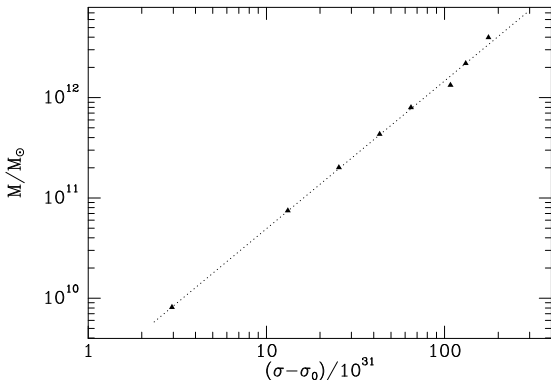


FIG. 1.— Distribution of gas mass as function of entropy index, σ , for the Hydra cluster. Triangles show observational results from David et al. (2001). The line shows the best-fitting relation of the form given by equation (5). The units of σ are $\text{cm}^4 \text{g}^{-2/3} \text{s}^{-2}$.

function of the gas density. Thus cooling is most important in the cluster center and the continued compression of the gas there leads to a gas flow inwards. On the way in the gas loses gravitational potential energy. Other ways in which the gas may gain or lose energy include the conversion of kinetic energy of the gas flow and thermal conduction. All these processes make the equation of energy difficult to solve.

In fact, we can formulate an equivalent, but considerably simpler set of equations to describe the evolution of the cluster gas. For this, we consider a variant of the entropy of the gas defined as

$$S = Nk_B \left[\text{const.} + \frac{3}{2} \ln(p\rho^{-5/3}) \right], \quad (2)$$

where N is the number of gas atoms, k_B is the Boltzmann constant and p is the gas pressure. Here we have assumed a monatomic gas with adiabatic index equal to $5/3$. Entropy itself is somewhat cumbersome to work with and so we define the ‘entropy index’ as

$$\sigma = p\rho^{-5/3}. \quad (3)$$

Entropy and therefore the entropy index does not change for adiabatic compression of the gas. If we further assume that all gas motions are subsonic, i.e. the kinetic energy of the gas is small compared to other forms of energy, and that thermal conduction is suppressed, then the only process changing the gas entropy is radiation. It is straightforward to show that

$$\dot{\sigma} = -\frac{2}{3}\rho^{1/3}\Lambda(T). \quad (4)$$

We will see in the following that using this simple expression instead of the conservation of energy equation leads to an analytical solution of the hydrostatic equations.

2.2. Entropy Index – Mass Relationship

Kaiser & Binney (2003) pointed out that in the Hydra cluster the cumulative mass of gas with an entropy index less than a given σ is a simple power-law function of σ itself, i.e.

$$M(<\sigma) = A(\sigma - \sigma_0)^\epsilon, \quad (5)$$

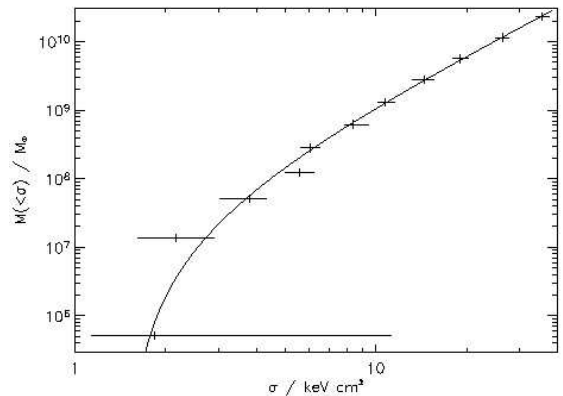


FIG. 2.— Distribution of gas mass as function of entropy index, σ , for the Virgo cluster. Data points show observational results from Matsushita et al. (2002). The line shows the best-fitting relation of the form given by equation (5).

where A is a constant and σ_0 is the entropy index of the gas with the lowest entropy in the entire atmosphere located at the center of the cluster. Figure 1 shows how closely a power-law of this form fits the observations. Another example is the Virgo cluster (see Figure 2; Kaiser 2003). More clusters for which a similar relation may very well provide an excellent fit to the data can be found in the contributions to these proceedings by Horner et al. and Donahue.

A parcel of gas with a given entropy index at a specific time t has a well-defined mass. This mass will not change under the influence of radiative cooling. Therefore at time $t + dt$ the parcel of gas will have a lower entropy index, but the mass of the gas in the parcel with this new value of σ will not have changed. Thus we can suspect that the power-law relation between $M(<\sigma)$ and $\sigma - \sigma_0$ may persist under the influence of radiative cooling. This was in fact confirmed by Kaiser & Binney (2003, see also section 4) by numerical integration of the hydrodynamical equations. An analytical argument for this behavior will be presented in a forthcoming paper.

3. Analytical Model for Evolving Cooling Flows

We use equation (4) giving the rate of change of the entropy index to replace the conservation of energy equation. The conservation of momentum equation remains unchanged, i.e.

$$\frac{\partial p}{\partial r} = -\rho \frac{\partial \Phi}{\partial r} = -\left(\frac{p}{\sigma}\right)^{3/5} \frac{\partial \Phi}{\partial r}, \quad (6)$$

where Φ is the gravitational potential. As usual we assume that Φ is generated by the dark matter halo of the cluster and remains unchanged by the motion of the gas.

We now assume a cooling function $\Lambda(T)$ appropriate for radiative cooling due to pure bremsstrahlung. This implies $\Lambda(T) \propto \sqrt{T}$. Clearly this is a strong simplification as it neglects any cooling due to line emission. Even for a gas of vanishing metallicity line emission will change the cooling function significantly for gas with temperatures below about 2 keV. However, above 2 keV this approximation of the cooling function is reasonable. Due to the very efficient cooling at low temperatures most of

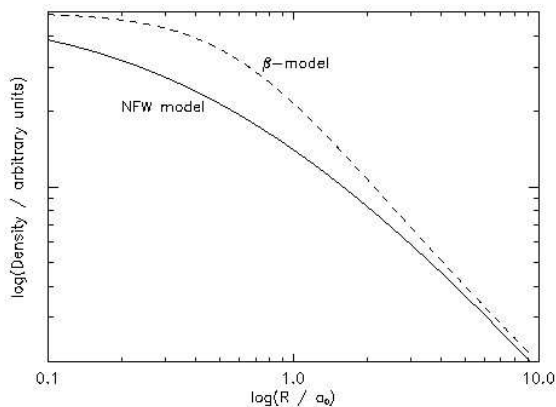


FIG. 3.— Density distribution of the cluster gas in the analytical model for two different gravitational potentials.

the cluster gas will spend only very little time with a temperature below this threshold. With this we find

$$\dot{\sigma} \propto p^{2/5} \sigma^{1/10}. \quad (7)$$

Clearly the rate of entropy loss is only a weak function of the changing gas properties. Therefore we set $p^{2/5} \sigma^{1/10} \approx \text{const.}$ Apart from the factor $\sigma^{1/10}$, this is equivalent to the assumption of isobaric cooling roughly consistent with the fact that the cooling timescale is long compared to the dynamical timescale of the gas.

Our final assumption in order to facilitate an analytical solution is that of a uniform entropy index, σ_i , of all the gas throughout the cluster atmosphere at $t = 0$. Again this may well be a gross oversimplification of the real situation as the cluster gas is presumably accumulated from a number of dark matter subhalos merging to form the final cluster halo. There is no reason why the gas contained in these subhalos should have the same entropy. However, any heating process affecting the entire cluster will eventually lead to a uniform entropy throughout the cluster atmosphere. Whether such a comprehensive heating phase has taken place or not is of course not clear. However, the assumption is crucial to obtaining an analytical solution.

With these assumptions we can solve equations (4) and (6) to find

$$\sigma = \left(\sigma_i^{8/5} - c_0 \Phi t \right)^{5/8} \quad (8)$$

and

$$p = \left[(\sigma_i - \sigma) \frac{c_1}{t} \right]^{5/2}. \quad (9)$$

Here, c_0 and c_1 are constants and the other thermodynamic properties of the gas may be derived from the above two expressions by assuming ideal gas conditions. Also note that the solution does not depend on the form of the gravitational potential Φ .

Figures 3 and 4 illustrate the density and temperature distributions in the cluster atmosphere predicted by the analytical model. As expected, assuming a β -profile for the gravitational potential results in the formation of a uniform density core while a NFW-profile (Navarro et al. 1996) results in a steeper central density distribution.

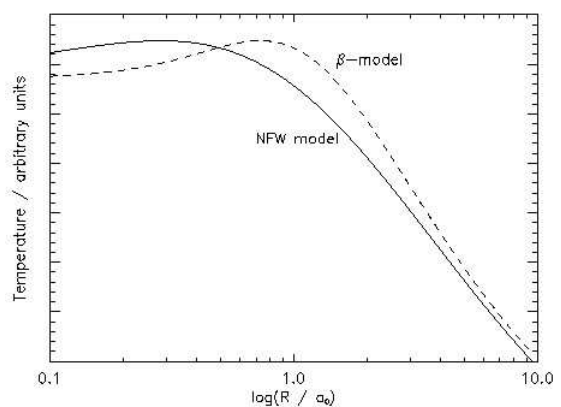


FIG. 4.— Temperature distribution of the cluster gas in the analytical model for two different gravitational potentials.

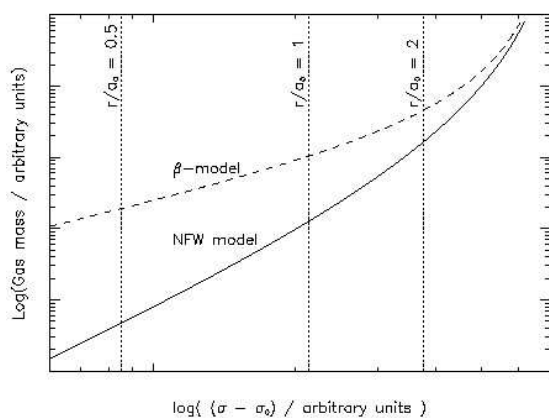


FIG. 5.— Cumulative mass, $M(< \sigma)$, as a function of entropy index as predicted by the analytical model. The vertical lines show the location of the gas within the cluster for the model with a gravitational potential described by the β -profile.

The temperature distributions show an off-center peak for both distributions, but the β -profile also results in a constant temperature core. This general shape with a constant temperature followed by a peak further out is reminiscent of the generic temperature profile found empirically by Allen et al. (2001) studying a sample of cooling flow clusters.

Finally, Figure 5 illustrates that, over a large range in entropy index and radius, the analytical model predicts a power-law relation between the gas mass and the entropy index as described by equation (5). The slope of the power-law, ϵ , in the inner part of the relation is roughly equal to 3 for the NFW-profile and 3/2 for the β -profile independent of the value of β . Given that the relation steepens for increasing radius, the latter value is closer to the slopes (Hydra: $\epsilon \sim 1.5$, Virgo: $\epsilon \sim 2.3$) found empirically from the data shown in Figures 1 and 2.

4. Numerical Approach

The results from the analytical model presented above may arise as a direct consequence of the great simplifications we employed in deriving this solution. In order to test this possibility, we have also solve equations (4) and (6) by numerical integration. Without the assumptions

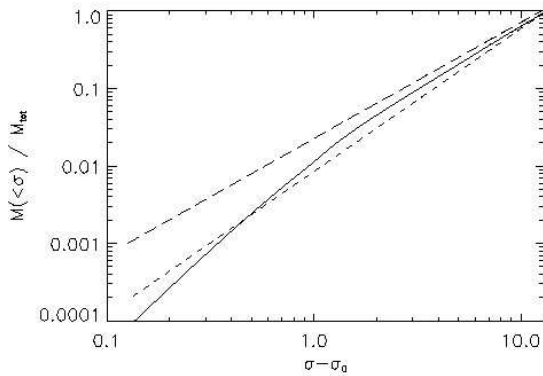


FIG. 6.— Entropy index – mass relation from the numerical solution. The long-dashed line shows the relation at the beginning of the integration approximating the observational data obtained in the Hydra cluster (see Figure 1). The relation at the end of the calculation when the central entropy index approaches zero is shown by the solid line. The short-dashed line shows the power-law relation of the form of equation (5) that provides the best approximation to the solid line.

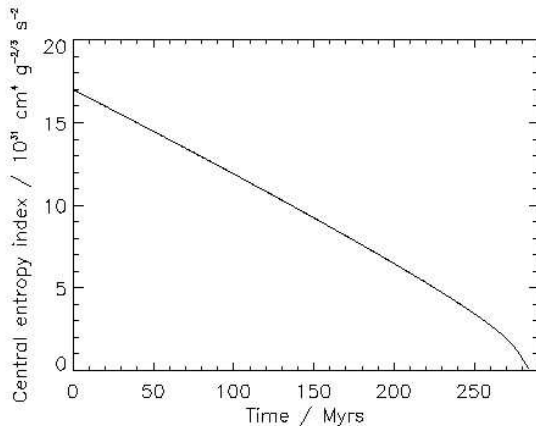


FIG. 7.— The evolution of the entropy index of the gas at the cluster center in the numerical solution.

we made above, we need a third equation to close the system. Here we use a variant of the equation of mass conservation in the form

$$\frac{\partial M}{\partial r} = \frac{\partial M}{\partial \sigma} \frac{\partial \sigma}{\partial r} = 4\pi r^2 \rho. \quad (10)$$

We start the integration with the gas distribution derived from the X-ray observations of the Hydra cluster given in David et al. (2001). These data provide us with the initial values for the derivative $\partial M/\partial \sigma$. We then use equation (4) to evolve the entropy index through a small timestep dt . We calculate the new derivative using the fact that the mass of gas with a given entropy index does not change (see section 2.2). Now it is possible to numerically solve equations (4) and (10). We then repeat the whole procedure for the next timestep. The numerical solution also includes a more realistic cooling function incorporating line emission. Details of the calculations are given in Kaiser & Binney (2003).

We continue the calculation until the entropy index and the temperature of the gas at the cluster center are equal to zero. Figure 6 demonstrates that the entropy

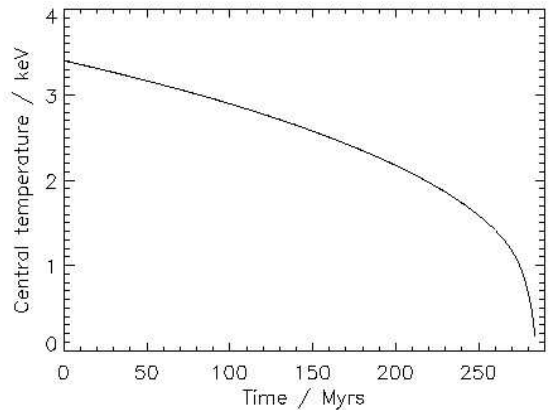


FIG. 8.— The evolution of the temperature of the gas at the cluster center in the numerical solution.

index – mass relationship continues to be well approximated by a power-law throughout the calculation. The inclusion of radiative cooling due to line emission does not change this relationship.

The entropy evolution is strongest at the cluster center as the pressure is highest there. Therefore we expect the deviations of the entropy evolution from a strictly linear behavior to be strongest in the cluster center. Figure 7 shows the evolution of the entropy index of the gas at the cluster center. Except for very late times the graph shows that this evolution is linear with time. Thus we conclude that our assumption for the analytical model of a linear evolution of the entropy index throughout the cluster is a good approximation to the real situation.

5. Why We Do Not Detect Cold Gas in Clusters

A further prediction of the model in both its analytical or numerical form is the strongly accelerating evolution of the gas temperature in the cluster center. Figure 8 shows the evolution of the gas temperature at the cluster center. By construction this is the coldest gas in the entire cluster. The temperature decrease only gradually for a few 10^8 years to about 2 keV. Only then the temperature drops rapidly to below 1 keV. This rapid drop is enhanced by the increasing importance of cooling due to line emission at about this temperature, but the drop would also occur in the absence of line cooling. The important point is that the lowest gas temperature in the cluster exists only for a very short fraction of the cluster evolution timescale. Furthermore, the volume of gas with temperatures below 1 keV or even 2 keV is small. In other words, the total emission measure of gas with temperatures below a given low threshold is only a small fraction of the total emission measure of the cluster.

Consider a sample of clusters with a given lower flux limit. Assume that every cluster in the sample has the same properties as the Hydra cluster at $t = 0$, but is observed at a random time in its evolution. We can then use the numerical model to estimate the fraction f of clusters in this sample which contain a detectable amount of gas at temperatures below a given threshold. Figure 9

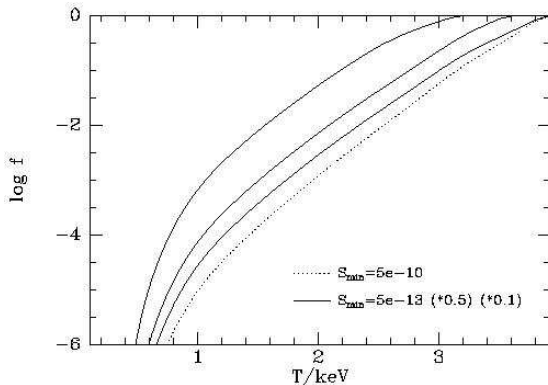


FIG. 9.— The fraction f of clusters in a flux-limited sample that contain a detectable amount of gas with temperatures below a threshold T as a function of T . The lines show the fraction f for various flux limits used to detect the cold gas in follow up observations to the original survey. Dotted line: The same flux limit as the original survey. Solid lines from bottom to top: 1000, 2000 and 10^4 times lower flux limits compared to the original survey.

presents the results of such an estimation. Even if the original survey is followed up by much more sensitive observations to search for cold gas, the fraction of clusters with detectable amounts of gas with temperatures below 1 keV is very small. It is therefore not surprising that only very few clusters containing cold gas have been found.

6. Summary

In this contribution we studied the evolution of the hot gas in galaxy clusters under the influence of radiative

cooling. By concentrating on the entropy of the gas, we were able to find an analytical solution for the relevant hydrodynamical equations. This solution assumes that

- the radiative cooling is due to pure bremsstrahlung.
- the evolution of the entropy index of a given parcel of gas is linear in time.
- the entropy index has a uniform value throughout the cluster atmosphere at $t = 0$.

With the help of a numerical integration of the same basic equations we showed that all of these assumptions are reasonable.

Both the analytical model and the numerical solution predict a simple power-law relationship between the entropy index and the gas mass consistent with observations. The accelerating temperature evolution of the model can explain why we do not observe many clusters with very cold gas at their centers. However, this result depends crucially on the triggering of a heating mechanism for the cluster gas at the time when the central temperature in the cluster approaches very low values. Without such a heating mechanism, this model encounters the usual problem that cold gas would rapidly accumulate at the cluster center. How the required heating mechanism works is not clear. However, the arrival of cold gas at the cluster center at the time when the mechanism must be triggered may point to a close connection between these two events.

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