Proceedings of *The Riddle of Cooling Flows in Galaxies and Clusters of Galaxies*: E44
May 31 – June 4, 2003, Charlottesville, Virginia, USA
Ed. T. H. Reiprich, J. C. Kempner, & N. Soker

Simulations of Cooling Flow Clusters with Thermal Conduction

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Using a new TreeGRAPE+SPH code, we have performed simulations to test the effects of thermal conduction on the physics of the Intracluster Medium (ICM), and in particular its viability as a mechanism to suppress cooling flows in massive clusters. Since the addition of thermal conduction into SPH is not a straightforward proposition, we describe a new numerical scheme and some tests aimed at untangling numerical effects from physical ones. Although some computational problems remain, we claim that the presence of conduction, even with the simplest approximation of a global conductivity, can substantially inhibit or delay the onset of a runaway cooling flow in a sufficiently massive cluster. We show that models with radiative cooling and thermal conduction are more similar to those without either one than they are to models that include cooling without conduction and generate a cooling flow. Moreover, the degree to which the cooling flow is inhibited can be adjusted by simply changing the value of the conductivity. We conclude by speculating on the the next stage approximations, which will include a stochastic treatment of magnetic fields and self-consistent galaxy formation and feedback. Although these processes clearly play a major role in the ICM, we see no indication that their inclusion will affect our basic conclusion that thermal conduction can be effective in decreasing or stopping cooling flows.

1. Introduction

The applicability of thermal conduction as a mechanism to suppress cooling flows has been vigorously debated over the last few years, without, as of yet, a clear consensus. In particular, the degree to which the conductivity is inhibited from the classical Spitzer value by magnetic fields, has been a subject of investigation from both, physical first principles and an observational perspective. While several studies have lead to suppression factors around 3–5 below Spitzer (e.g. Narayan & Medvedev 2001; Gruzinov 2002; Zakamska & Narayan 2003), other lines of approach have produced values of order 10^2–10^3 (e.g. Chandran & Cowley 1998; Ettori & Fabian 2000), and different results in between. Various attempts at the very difficult task of semi-analytic modeling of a conductive ICM have not achieved convergence either.

Because of the difficulties mentioned above, we believe that the inclusion of heat conduction in a full 3-dimensional simulation of a cooling flow cluster is very important. For that purpose, we have developed a new parallel, gravity-hydrodynamic code that is particularly geared towards simulations of individual bound structures such as galaxies and clusters. The most salient features of the program are:

- The gravitational computations are done by a combination of a modified treecode with the special purpose hardware GRAPE. The standard treecode algorithm, and the parallelization scheme, had to be modified extensively to accommodate the requirements of the GRAPE boards in a parallel environment. The gravitational module, in fact, is very flexible, being able to run in tree or direct summation mode, in serial or parallel, with any GRAPE version higher than 3, or without it altogether.

- Major modifications to the SPH part are the inclusion of conduction and an integration scheme in which particles have individual SPH time steps (Courant) while remaining on a global time step for gravity, since parallel GRAPE is ill suited for individual time steps.

- When conduction is present, cooling is computed in an explicit way in the same sub-cycle as conduction, instead of the usual implicit scheme. This is done in a third *layer*, so that many cooling-conduction steps can be done within a single adiabatic Courant step, of which in turn there can be many within one global, dynamical step.

- The program accommodates both cosmological vacuum and periodic boundary conditions through the Ewald method, although that is *very* slow when using GRAPE because computations have to be done in software for every pairwise interaction. Thus cluster simulations are done by first running low resolution, dark-matter-only cosmological simulations with Ewald boundary conditions, and then resampling the areas of interest using a tree traversal for the construction of a multi-mass model.

- Performance, in terms of speed and parallelization efficiency, is comparable to the bare-bones benchmarks of the GRAPE system on which it was tested, which is encouraging for a complex program.

- The code was put through a large number of tests, including all the industry standards (e.g. Thacker et al. 2000) such as the shock tube, collapse of isothermal sphere, rotating cloud with radiative cooling, galaxy formation, cold clumps in hot medium and the Santa Barbara Cluster Test (Frenk et al. 1999).

2. Thermal Conduction in SPH

Although it would at first appear that the diffusive nature of the the SPH algorithm is well suited to handle a transport process, the computation of the divergence of the heat flux ($\nabla \cdot \mathrm{q} = -\nabla \cdot (\kappa \nabla T)$) requires the evaluation of second derivatives, which are very unreliable in SPH. A workaround was proposed by Cleary & Monaghan (1999), where by using an integral approximant, the conductive term added to the energy equation is given by

$$\left.\frac{du_i}{dt}\right|_{cond} = \sum_j \frac{m_j}{\rho_i \rho_j} \kappa_{\rm eff}(T_i - T_j) F(\mathbf{r}_{ij}, h_{ij}), \quad (1)$$

where $F(\mathbf{r}_{ij}, h_{ij}) = \mathbf{r} \cdot \nabla_i W(\mathbf{r}_{ij}, h_{ij})/r$ and $\kappa_{\rm eff} = 4\kappa_i\kappa_j/(\kappa_i + \kappa_j)$ is the symmetrized conductivity. The conductivity of each particle is the classical Spitzer conductivity (Spitzer 1962) $\kappa_S(T) \simeq 5.0 \times 10^{-7} T^{5/2}\ \mathrm{erg\,s^{-1}\,cm^{-1}\,K^{-1}}$, multiplied by a *fudge* parameter (f_S) that captures the average suppression due to magnetic fields.

Although algorithmically simple there are a number of complications associated with it:

- The characteristic time scales ($\propto \frac{c_v \rho_i h_i^2}{\kappa_i}$) can become very short, forcing integration with a very small time step. In fact, using numbers typical of cluster cores one can get integration steps of order 10^4 yrs, requiring 10^6 steps in a Hubble time.

- The implicit integration scheme normally used to treat the same problem when associated with radiative cooling can not be used, because unlike the cooling term, conduction requires a loop over all neighbors.

- As consequence of the above, it requires a complex nested integration scheme, which is *very* costly in computational time and less stable than the usual implicit schemes.

- Because the conductive flux goes as $T^{7/2}$, the system is extremely sensitive to local temperature gradients, and becomes susceptible to numerical instabilities due to noise in the temperature field.

- The use of a constant *fudge* factor is a questionable approximation. In all likelihood, the degree to which conductivity is suppressed from the classical Spitzer value is not constant in either time or space, nor is it isotropic, as it is controlled by the tangled magnetic fields in the system.

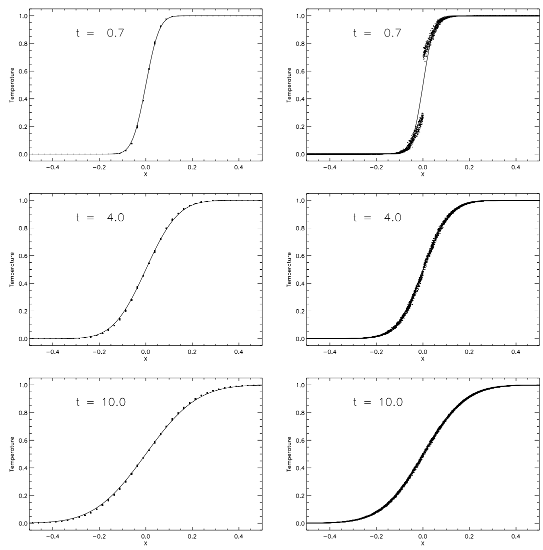

FIG. 1.— Conduction in a slab with an initial step discontinuity at $x = 0$ shown at 3 different times. The left panels show a system with gridded initial conditions, and the right panels are for an initial random, thermalized distribution. Time, distance and temperature in arbitrary units.

- The higher the resolution of a run, the better the temperature field is resolved and exhibits more substructure. This, in turn, creates steeper local gradients which exacerbate all of the above problems. This is, of course, particularly pernicious, as we require very large runs in order to be able to resolve the cluster cores and expose the cooling flow.

We have performed a number of tests under controlled conditions, and the algorithm behaves very well. Figure 1 shows the evolution of the temperature in a one dimensional test fitted with an analytical solution, and the residuals (not shown) are always less than 2%.

3. Simulations With Conduction

We have performed a number of runs for the same cosmological initial conditions, generated in a constrained random realization so as to result in a large cluster ($M = 10^{15} M_\odot$) at the center. The cosmological model used was a standard ΛCDM with $\Omega_{DM} = 0.27$, $\Omega_B = 0.03$ and $\Omega_\Lambda = 0.7$. The initial cosmological volume simulated with fully periodic boundary conditions (dark matter only) was 192 Mpc^3, and the resulting cluster was re-simulated with gas included using vacuum boundary conditions. The simulation was started at a redshift of 20, and the cluster underwent its final major merger at $z = 0.8$. The final cluster had 2×120,000 particles in the virial region. We ran this same scenario using adiabatic physics only, including cooling and including conduction with various values of the fudge parameter (a value of 0.15 was used for the profiles shown in Figure 3).

The choice of initial conditions (i.e., the mass of our fiducial cluster) was guided by the assumption that conduction is more likely to be effective for massive clusters. A simple timescale analysis indicates a threshold at around $0.7 \times 10^{15} M_\odot$ (the right dotted line in Figure 2)

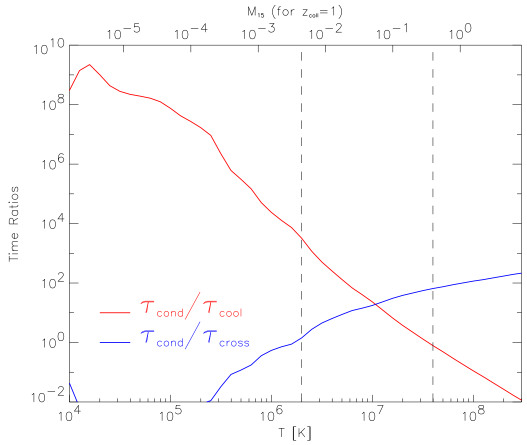

FIG. 2.— Ratios of various time scales: the cooling time, $\tau_{cool} \equiv \frac{3}{2}\frac{k_b T}{n\Lambda(T)}$, where n is the gas density and $\Lambda(T)$ is the standard cooling function; the conduction time scale $\tau_{cond} \equiv \frac{3}{2}\frac{n k_b R^2}{\kappa(T)}$, where R is the characteristic length scale, $\kappa(T)$ is the effective conductivity and k_b is the Boltzmann constant; τ_{uni} is the Hubble time. The vertical dashed lines are simply markers for the points where τ_{cool}/τ_{uni} (left) and τ_{cond}/τ_{cool} (right) are unity. The line at $M = 7\times10^{14} M_\odot$ marks the transition from the cooling-dominated (to the left) to the conduction dominated (to the right) regime.

above which conduction (with a $T^{7/2}$) dependence dominates, and below which cooling will tend to win.

The comparison of the runs is summarized in Figure 3, which shows the radial profiles for the gas density, temperature, infall radial velocity and entropy for the three types of runs. They clearly indicate that the inclusion of heat conduction acts towards suppressing the cooling flow, and the profiles of the conductive run resemble those of the adiabatic one more than they do that of the cooling only run. This is shown explicitly in the plot of the mass accretion rate (Figure 4). It should be noted, however, that these profiles are for the entire virial radius of the cluster, and that the cooling flow region is sampled by only the three innermost points of the profile. With the higher resolution runs being currently performed we should be able to make these claims with more confidence.

4. Conclusions

Despite some outstanding numerical issues, we have succeeded in running full, 3-dimensional simulations of a cooling flow cluster, and when conduction is included the cooling flow is suppressed by a factor of 5 or more. Although further improvements are needed before a robust determination of the structure of a conductive core can be made, the current results provide strong evidence that heat conduction, at the level of 0.15–0.3 Spitzer can in fact substantially inhibit a cooling flow.

The main difficulty we have dealt with in this work are the instabilities, physical and numerical created by the inclusion of conduction, which have forced us to perform relatively low resolution runs, where the cooling flow is not as well resolved and can in fact be artificially suppressed due to gravitational softening. We are currently working on several strategies to cope with this problem, some purely numerical and also a scheme to decouple the electron temperature from the ion temperature, which would allow the system to react faster to changing gradients. The most important next step, is the inclusion of the effect of magnetic fields on the suppression factor, which so far has been set at an arbitrary

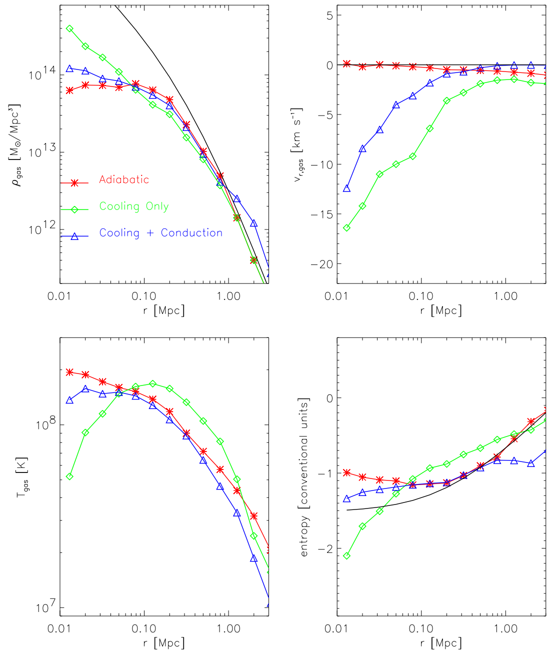

FIG. 3.— Comparison of gas radial profiles for a cooling flow cluster, simulated with adiabatic physics only, including cooling, and including cooling and conduction. The profiles have been computed with 15 logarithmic bins of with 0.2 decades, and the plots have been cut off at the virial radius. The solid line in the density plot corresponds to an NFW profile (normalized to the baryon fraction); the solid line in the entropy plot corresponds to the prediction of a *pre-heated* model with a power law form for the adiabat.

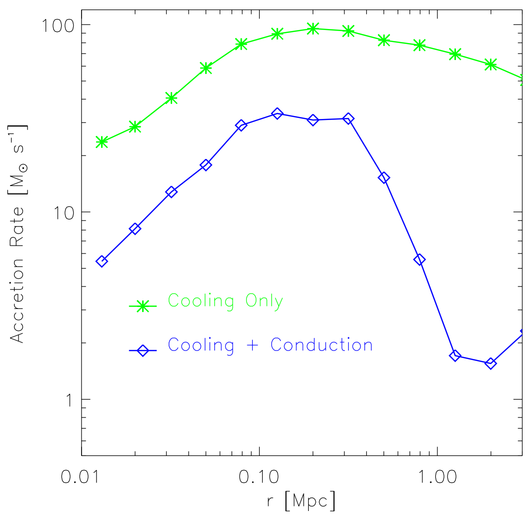

FIG. 4.— Mass accretion rates with and without conduction. The rate is defined as $\dot{M} = 4\pi r^2 \rho v_r$.

level. Although we are not including the *dynamic* effects of magnetic fields, a stochastic approach is sufficient as far as conduction is concerned. Finally, the inclusion of galaxy formation and feedback is essential for a complete picture to be formed, particularly when conduction is considered, as point sources of heat create temperature gradients around themselves which may give a substantial contribution to the overall conductive efficiency.

This work was done as part of the author's doctoral dissertation at the Department of Physics and Astronomy, The Johns Hopkins University, Baltimore, MD, under the supervision of Prof. Colin Norman. All numerical work was performed at the computational facilities of the National Astronomical Observatory, Tokyo, Japan.

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