HW2, Astronomy 542, Spring 2009.

Due Tuesday Feb.16

1) A star of radius R emits blackbody radiation $I_\nu = B_\nu(T)$ from its surface. Derive, *by integrating over the intensity*, the monochromatic energy flux and frequency integrated energy density a distance $d \gg R$ from the star. (Hint: this problem is all about solid angles). Frequently the energy density of starlight is written as $u = aT^4W$, where W is the dilution constant. What is W? Compare the energy density of starlight $d = 1$ pc away from a single solar type star, to the energy density of the Cosmic Microwave Background.

2) A certain gas emits *thermally* at the rate $P(\nu)$ (power per unit volume per unit frequency). A spherical cloud of this gas has radius R, temperature T, and is a distance $d \gg R$ away from Earth.

(a) Assume that the cloud is optically *thin* for parts (a)-(c). What is the brightness of the cloud, as measured on Earth, as a function of angular distance θ from the cloud center ($\theta = 0$) to the cloud edge ($\theta \simeq R_s/d$). Note that in the $d >> R$ limit, all rays through the cloud are approximately parallel.

(b) What is the flux F_ν at Earth from the entire cloud?

(c) How does the measured brightness temperature compare with the cloud's temperature T? (Hint: for a thermally emitting region, the transfer equation gives $I_\nu \simeq B_\nu(T)(1 - e^{-\tau_\nu})$).

Parts (d)-(f). Repeat parts (a)-(c) for an optically thick cloud.

(g) Based on your answers to parts (b) and (e), discuss qualitatively how the measured flux depends on cloud radius.

3) In class, we derived the radiation transfer equation expressed in terms of the brightness temperature. Let a background source of brightness temperature $T_{B,0}$ pass through a cloud of temperature T and total optical depth τ (at some frequency).

(a) Derive the change in brightness temperature due to the cloud, ΔT_B. This can be interpreted as either absorption ($\Delta T_B < 0$) or emission ($\Delta T_B > 0$) relative to the continuum.

(b) Evaluate the result of part (a) in the limits of small and large optical depth.

4) The CN molecule has a transition from the $J = 1$ rotational state to the $J = 0$ rotational state with wavelength $\lambda = 2.64$ mm and Einstein $A_{10} \simeq 10^{-5}$ s^{-1}. The degeneracy of a rotational state J is $g_J = 2J + 1$.

a) Compute the other two Einstein coefficients B_{01} and B_{10}.

b) Assume that the gas translational motions have temperature $T_{\rm tran} = 10$ K and that the level populations are set by interaction with the Cosmic Microwave Background, which is a radiation field decribed by a blackbody with temperature $T_{\rm cmb} = 2.7$ K. What is the ratio N_1/N_0, the ratio of the number of molecules in the $J = 1$ state to the $J = 0$ state?

Next, for parts (c)-(d), let the transfer equation for the CMB through the gas cloud of CN molecules be

$$\frac{dI_\nu}{ds} = j_\nu - \alpha_\nu I_\nu \tag{1}$$

where the first term is emission from the gas, and the second term represents absorption corrected by stimulated emission.

c) Using the assumption in part (b), how are j_ν and α_ν related? (Hint: use Kirchoff).

d) Using the result of part (c), simplify the above transfer equation by eliminating j_ν in favor of α_ν. Does the gas cloud modify the CMB spectrum near $\lambda = 2.64$ mm so as to produce an absorption line or emission line?

4) Dust is produced in the winds of cool stars - Asymptotic Giant Branch and post-AGB stars. Dust is also produced in Wolf-Rayet stars and Supernovae, but the yields are less well known and probably not as significant. Dust may be produced in cold quiescent molecular clouds, but even less is known about that, so let's consider the dominant source.

a) Calculate the equilibrium temperature of a dust grain if size a at radius r. Assume that the wind is optically thin, so the mean intensity is simply determined by the radius and temperature of the star $R_\star$ and $T_\star$. The grain is only heated and cooled radiatively, with opacity κ_ν. You may simplify the geometric dilution factor by assuming that $r >> R_\star$. You will find the need to evaluate the ratio of two integrals over Plank functions; the integrals can be cancelled if you assume that the absorptive efficiency Q is a power law $Q \propto \lambda^{-p}$.

b) AGB stars have effective temperatures of 2000-3000K. Dust sublimates above ∼1500K, so calculate the inner radius of the dust condensation zone relative to the stellar radius $R_\star \simeq 300 R_\odot$.

c) Assume that the wind is at its terminal velocity of 10-25 km/s (it accelerates to that velocity relatively close to the star). For simplicity, we'll just consider one grain size a with density $\rho_d \simeq 3 gcm^{-3}$. Write down the total dust mass loss rate $\dot{M}_d$ in terms of the wind velocity v, dust number density $n_d(r)$, and grain properties a and ρ_d.

d) Write down the integral of grain emission from r_{min} to r_{max} to obtain the total L_ν (in terms of $\dot{M}_d$, v, a, ρ_d, κ_ν, and the dust temperature). Now put it all together: using the dust temperature T_d from (a), $\kappa_\nu \propto \nu^p$ (pick a reasonable normalization constant from the infrared parts of plots in the textbook), and r_{max} relatively large, calculate the spectral energy distribution for a dusty cool star wind. You will need to pick a wind velocity v in the range above, and a dust mass loss rate. Total mass loss rates for AGB stars are approximately 10^{-5}M$_\odot$/yr, and the gas-to-dust ratio is 200, a bit higher than in the average ISM.

For realism, add in the black-body from the stellar photospheric emission (ignoring the fact that it is in reality extincted by its own dusty wind). Show the spectra for one choice of $R_\star$ and $T_\star$, for r_{min} equal to the condensation radius, and 10 and 100 times larger. In the case of post-AGB stars, the wind has terminated, but the material continues to coast outwards, so the inner edge will be at $r_{min} > r_{cond}$. Do it once with the dust opacity power law index p=2, as in the diffuse ISM, and p=1, which is closer to true in AGB stars.

e) Why might the dust opacity by flatter (smaller p) in an AGB star wind than in the diffuse ISM?